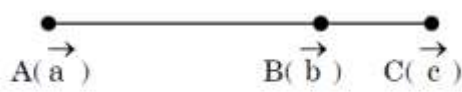


6.	<p>6. $\int_0^{\frac{\pi}{6}} \sec^2(x - \frac{\pi}{6}) dx$ is equal to :</p> <p>(a) $\frac{1}{\sqrt{3}}$ (b) $-\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) $-\sqrt{3}$</p>	
Ans	(a) $\frac{1}{\sqrt{3}}$	1
7.	<p>7. The sum of the order and the degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = \sin y$ is :</p> <p>(a) 5 (b) 2 (c) 3 (d) 4</p>	
Ans	(c) 3	1
8.	<p>8. The value of p for which the vectors $2\hat{i} + p\hat{j} + \hat{k}$ and $-4\hat{i} - 6\hat{j} + 26\hat{k}$ are perpendicular to each other, is :</p> <p>(a) 3 (b) -3 (c) $-\frac{17}{3}$ (d) $\frac{17}{3}$</p>	
Ans	(a) 3	1
9.	<p>9. The value of $(\hat{i} \times \hat{j}) \cdot \hat{j} + (\hat{j} \times \hat{i}) \cdot \hat{k}$ is :</p> <p>(a) 2 (b) 0 (c) 1 (d) -1</p>	
Ans	(d) -1	1
10.	<p>10. If $\vec{a} + \vec{b} = \hat{i}$ and $\vec{a} = 2\hat{i} - 2\hat{j} + 2\hat{k}$, then \vec{b} equals :</p> <p>(a) $\sqrt{14}$ (b) 3 (c) $\sqrt{12}$ (d) $\sqrt{17}$</p>	
Ans	(b) 3	1
11.	<p>11. Direction cosines of the line $\frac{x-1}{2} = \frac{1-y}{3} = \frac{2z-1}{12}$ are :</p> <p>(a) $\frac{2}{7}, \frac{3}{7}, \frac{6}{7}$ (b) $\frac{2}{\sqrt{157}}, -\frac{3}{\sqrt{157}}, \frac{12}{\sqrt{157}}$ (c) $\frac{2}{7}, -\frac{3}{7}, -\frac{6}{7}$ (d) $\frac{2}{7}, -\frac{3}{7}, \frac{6}{7}$</p>	
Ans	(d) $\frac{2}{7}, -\frac{3}{7}, \frac{6}{7}$	1

12.	<p>12. If $P\left(\frac{A}{B}\right) = 0.3$, $P(A) = 0.4$ and $P(B) = 0.8$, then $P\left(\frac{B}{A}\right)$ is equal to :</p> <p>(a) 0.6 (b) 0.3 (c) 0.06 (d) 0.4</p>	
Ans	(a) 0.6	1
13.	<p>13. The value of k for which $f(x) = \begin{cases} 3x + 5, & x \geq 2 \\ kx^2, & x < 2 \end{cases}$ is a continuous function, is :</p> <p>(a) $-\frac{11}{4}$ (b) $\frac{4}{11}$ (c) 11 (d) $\frac{11}{4}$</p>	
Ans	(d) $\frac{11}{4}$	1
14.	<p>14. If $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $(3I + 4A)(3I - 4A) = x^2I$, then the value(s) x is/are :</p> <p>(a) $\pm\sqrt{7}$ (b) 0 (c) ± 5 (d) 25</p>	
Ans	(c) ± 5	1
15.	<p>15. The general solution of the differential equation $x dy - (1 + x^2) dx = dx$ is :</p> <p>(a) $y = 2x + \frac{x^3}{3} + C$ (b) $y = 2 \log x + \frac{x^3}{3} + C$ (c) $y = \frac{x^2}{2} + C$ (d) $y = 2 \log x + \frac{x^2}{2} + C$</p>	
Ans	(d) $y = 2 \log x + \frac{x^2}{2} + C$	1
16.	<p>16. If $f(x) = a(x - \cos x)$ is strictly decreasing in \mathbb{R}, then 'a' belongs to</p> <p>(a) $\{0\}$ (b) $(0, \infty)$ (c) $(-\infty, 0)$ (d) $(-\infty, \infty)$</p>	
Ans	(c) $(-\infty, 0)$	1

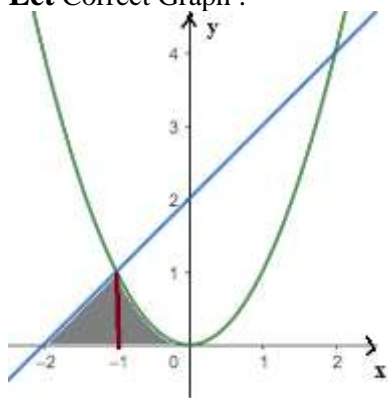
17.	<p>17. The corner points of the feasible region in the graphical representation of a linear programming problem are (2, 72), (15, 20) and (40, 15). If $z = 18x + 9y$ be the objective function, then :</p> <p>(a) z is maximum at (2, 72), minimum at (15, 20)</p> <p>(b) z is maximum at (15, 20), minimum at (40, 15)</p> <p>(c) z is maximum at (40, 15), minimum at (15, 20)</p> <p>(d) z is maximum at (40, 15), minimum at (2, 72)</p>	
Ans	(c) z is maximum at (40, 15) and minimum at (15, 20)	1
18.	<p>18. The number of corner points of the feasible region determined by the constraints $x - y \geq 0$, $2y \leq x + 2$, $x \geq 0$, $y \geq 0$ is :</p> <p>(a) 2</p> <p>(b) 3</p> <p>(c) 4</p> <p>(d) 5</p>	
Ans	(a) 2	1
(Question Nos. 19 & 20 are Assertion-Reason based questions of 1 mark each)		
19.	<p>19. Assertion (A) : The range of the function $f(x) = 2 \sin^{-1} x + \frac{3\pi}{2}$, where $x \in [-1, 1]$, is $\left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$.</p> <p>Reason (R) : The range of the principal value branch of $\sin^{-1}(x)$ is $[0, \pi]$.</p>	
Ans	(c) Assertion is True, Reason is False	1
20.	<p>20. Assertion (A) : Equation of a line passing through the points (1, 2, 3) and (3, -1, 3) is $\frac{x-3}{2} = \frac{y+1}{3} = \frac{z-3}{0}$.</p> <p>Reason (R) : Equation of a line passing through points (x_1, y_1, z_1), (x_2, y_2, z_2) is given by $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$.</p>	
Ans	(d) Assertion is False, Reason is True	1
SECTION-B		
(Question nos. 21 to 25 are very short Answer type questions carrying 2 marks each)		
21.	<p>21. (a) A function $f : A \rightarrow B$ defined as $f(x) = 2x$ is both one-one and onto. If $A = \{1, 2, 3, 4\}$, then find the set B.</p> <p style="text-align: center;">OR</p> <p>(b) Evaluate :</p> $\sin^{-1}\left(\sin \frac{3\pi}{4}\right) + \cos^{-1}\left(\cos \frac{3\pi}{4}\right) + \tan^{-1}(1)$	
Ans	<p>(a) $f(1) = 2, f(2) = 4, f(3) = 6, f(4) = 8$</p> <p>$\therefore B = \{2, 4, 6, 8\}$</p> <p style="text-align: center;">OR</p> <p>(b) Required value = $\frac{\pi}{4} + \frac{3\pi}{4} + \frac{\pi}{4}$</p>	$1\frac{1}{2}$ $1\frac{1}{2}$

	$= \frac{5\pi}{4}$	$\frac{1}{2}$
22.	22. Find all the vectors of magnitude $3\sqrt{3}$ which are collinear to vector $\hat{i} + \hat{j} + \hat{k}$.	
Ans	Unit vector along $\hat{i} + \hat{j} + \hat{k}$ is $\frac{\hat{i}}{\sqrt{3}} + \frac{\hat{j}}{\sqrt{3}} + \frac{\hat{k}}{\sqrt{3}}$ Required vectors are $3\hat{i} + 3\hat{j} + 3\hat{k}$ and $-3\hat{i} - 3\hat{j} - 3\hat{k}$	1 $\frac{1}{2} + \frac{1}{2}$
23.	23. (a) Position vectors of the points A, B and C as shown in the figure below are \vec{a} , \vec{b} and \vec{c} respectively.  If $\vec{AC} = \frac{5}{4}\vec{AB}$, express \vec{c} in terms of \vec{a} and \vec{b} . OR (b) Check whether the lines given by equations $x = 2\lambda + 2$, $y = 7\lambda + 1$, $z = -3\lambda - 3$ and $x = -\mu - 2$, $y = 2\mu + 8$, $z = 4\mu + 5$ are perpendicular to each other or not.	
Ans	(a) According to question, $\vec{c} - \vec{a} = \frac{5}{4}(\vec{b} - \vec{a})$ $\therefore \vec{c} = \frac{5\vec{b}}{4} - \frac{\vec{a}}{4}$ OR (b) D.r.s. of lines are $\langle 2, 7, -3 \rangle$ and $\langle -1, 2, 4 \rangle$ Now $2 \cdot -1 + 7 \cdot 2 + -3 \cdot 4 = 0$ \therefore given lines are perpendicular	$1\frac{1}{2}$ $\frac{1}{2}$ 1 1
24.	24. If $y = (x + \sqrt{x^2 - 1})^2$, then show that $(x^2 - 1) \left(\frac{dy}{dx}\right)^2 = 4y^2$.	
Ans	$\frac{dy}{dx} = 2(x + \sqrt{x^2 - 1}) \left(1 + \frac{x}{\sqrt{x^2 - 1}}\right) = \frac{2(x + \sqrt{x^2 - 1})^2}{\sqrt{x^2 - 1}}$ $\sqrt{x^2 - 1} \frac{dy}{dx} = 2y$ $(x^2 - 1) \left(\frac{dy}{dx}\right)^2 = 4y^2$	$1\frac{1}{2}$ $\frac{1}{2}$

25.	25. Show that the function $f(x) = \frac{16 \sin x}{4 + \cos x} - x$, is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$.	
Ans	$f'(x) = \frac{16[4 + \cos x] \cos x + 16 \sin^2 x}{(4 + \cos x)^2} - 1$ $= \frac{\cos x (56 - \cos x)}{(4 + \cos x)^2}$ <p>in $\left(\frac{\pi}{2}, \pi\right)$, $\cos x < 0 \Rightarrow f'(x) < 0$</p> <p>$\therefore f(x)$ is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$</p>	<p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>
SECTION-C (Question nos. 26 to 31 are short Answer type questions carrying 3 marks each)		
26.	<p>26. Evaluate :</p> $\int_0^{\frac{\pi}{2}} [\log(\sin x) - \log(2 \cos x)] dx.$	
Ans	<p>Let $I = \int_0^{\pi/2} [\log \sin x - \log(2 \cos x)] dx = \int_0^{\pi/2} \log \left(\frac{\tan x}{2}\right) dx$</p> <p>Using property $\int_0^a f(x) dx = \int_0^a f(a-x) dx$</p> <p>We get, $I = \int_0^{\pi/2} \log \left(\frac{\cot x}{2}\right) dx$</p> <p>$\therefore 2I = \int_0^{\pi/2} \log \left(\frac{\tan x}{2} \times \frac{\cot x}{2}\right) dx = \int_0^{\pi/2} \log \left(\frac{1}{4}\right) dx$</p> <p>$2I = \log \left(\frac{1}{4}\right) x \Big _0^{\pi/2} = \frac{\pi}{2} \log \frac{1}{4}$</p> <p>$I = \frac{\pi}{4} \log \frac{1}{4}$ OR $-\frac{\pi}{2} \log 2$</p>	<p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>
27.	<p>27. Find :</p> $\int \frac{1}{\sqrt{x}(\sqrt{x}+1)(\sqrt{x}+2)} dx$	
Ans	<p>Let $I = \int \frac{dx}{\sqrt{x}(\sqrt{x}+1)(\sqrt{x}+2)}$</p>	

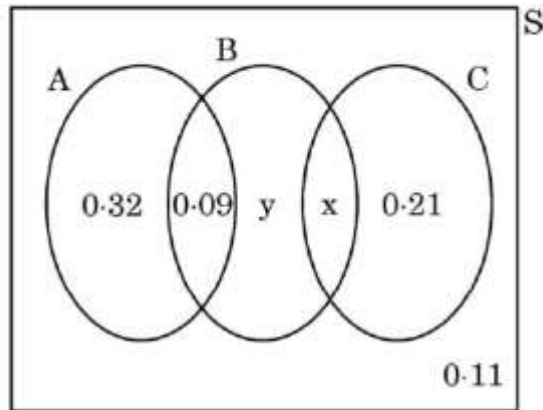
	<p>Let $\sqrt{x} = t$, $\frac{1}{2\sqrt{x}} dx = dt$</p> $\therefore I = 2 \int \frac{dt}{(t+1)(t+2)}$ $= 2 \int \left(\frac{1}{t+1} - \frac{1}{t+2} \right) dt$ $= 2[\log t+1 - \log t+2] + C$ $= 2[\log(\sqrt{x} + 1) - \log(\sqrt{x} + 2)] + C \text{ or } 2 \log \left(\frac{\sqrt{x} + 1}{\sqrt{x} + 2} \right) + C$	<p>$\frac{1}{2}$</p> <p>1</p> <p>1</p> <p>$\frac{1}{2}$</p>
28.	<p>28. (a) Find the particular solution of the differential equation $\frac{dy}{dx} + \sec^2 x \cdot y = \tan x \cdot \sec^2 x$, given that $y(0) = 0$.</p> <p style="text-align: center;">OR</p> <p>(b) Solve the differential equation given by $x dy - y dx - \sqrt{x^2 + y^2} dx = 0$.</p>	
Ans	<p>Let (a) Integrating factor = $e^{\int \sec^2 x dx} = e^{\tan x}$</p> <p>Solution is $ye^{\tan x} = \int \tan x \sec^2 x e^{\tan x} dx + C$</p> <p>Let $\tan x = t$ $\sec^2 x dx = dt$</p> $\therefore \int e^{\tan x} \tan x \sec^2 x dx = \int e^t t dt = e^t (t - 1)$ $\therefore ye^{\tan x} = e^{\tan x} (\tan x - 1) + C$ <p>$y(0) = 0$ gives $C = 1$</p> <p>Particular solution is $ye^{\tan x} = e^{\tan x} (\tan x - 1) + 1$ or $y = \tan x - 1 + e^{-\tan x}$</p> <p style="text-align: center;">OR</p> <p>(b) Given differential equation can be written as</p> $\frac{dy}{dx} = \frac{y}{x} + \sqrt{1 + \left(\frac{y}{x}\right)^2} \text{ ----- (i)}$ <p>Let $y = vx \Rightarrow \frac{dv}{dx} = v + x \frac{dv}{dx}$ substituting in (i)</p> <p>We get $v + x \frac{dv}{dx} = v + \sqrt{1 + v^2}$</p> $\Rightarrow \frac{dv}{\sqrt{1 + v^2}} = \frac{dx}{x}$ <p>Integrating both sides, we get</p> $\log \sqrt{1 + v^2} + v = \log x + \log C$ $y + \sqrt{x^2 + y^2} = C x^2$	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p>

	$= \frac{1}{\sin(a-b)} [\log \sec(x-b) - \log \sec(x-a)] + C$	1
SECTION-D		
(Question nos. 32 to 35 are Long Answer type questions carrying 5 marks each)		
32.	<p>32. A relation R is defined on a set of real numbers \mathbb{R} as</p> $R = \{(x, y) : x \cdot y \text{ is an irrational number}\}.$ <p>Check whether R is reflexive, symmetric and transitive or not.</p>	
Ans	<p>For reflexive $(1, 1) \notin R$ as 1^2 is rational (or any other counter example) R is not reflexive For symmetric Let $(x, y) \in R \therefore x \cdot y$ is an irrational number $\therefore (y, x)$ is an irrational number $\therefore (y, x) \in R$ $\therefore R$ is symmetric For Transitive $(1, \sqrt{2}) \in R, (\sqrt{2}, 2) \in R$ (or any other counter example) but $(1, 2) \notin R$ $\therefore R$ is not transitive</p>	<p>1 ½</p> <p>1 ½</p> <p>2</p>
33.	<p>33. (a) If $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$, find $(AB)^{-1}$.</p> <p style="text-align: center;">OR</p> <p>(b) Solve the following system of equations by matrix method :</p> $x + 2y + 3z = 6$ $2x - y + z = 2$ $3x + 2y - 2z = 3$	
Ans	<p>(a) $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}, B^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ $(AB)^{-1} = B^{-1}A^{-1}$ $A = 1(3) - 2(-1) - 2(2) = 3 + 2 - 4 = 1 \neq 0$ $\text{adj}(A) = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$ $A^{-1} = \frac{1}{1} \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$ $\therefore B^{-1}A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$</p>	<p>½</p> <p>1</p> <p>2</p> <p>½</p>

	$\therefore d = \frac{\sqrt{293}}{7}$ <p style="text-align: center;">OR</p> <p>(b) Equation of line AB is $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{6}$</p> <p>Let coordinates of required point on AB be $(2\lambda + 1, 3\lambda + 2, 6\lambda + 3)$ for some λ</p> <p>According to Question</p> $(2\lambda - 2)^2 + (3\lambda - 3)^2 + (6\lambda - 6)^2 = 14^2 \text{ gives } \lambda^2 - 2\lambda - 3 = 0$ <p>Solving we get $\lambda = 3$ and -1</p> <p>\therefore required points are $(7, 11, 21)$ and $(-1, -1, -3)$</p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>
35.	<p>35. Find the area of the region bounded by the curves $x^2 = y$, $y = x + 2$ and x-axis, using integration.</p>	
Ans	<p>Let Correct Graph :</p>  <p>x coordinates of point of intersection are $-1, 2$</p> $\text{Required area} = \int_{-2}^{-1} (x + 2) dx + \int_{-1}^0 x^2 dx$ $= \left. \frac{(x + 2)^2}{2} \right _{-2}^{-1} + \left. \frac{x^3}{3} \right _{-1}^0$ $= \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$	<p>1 ½</p> <p>½</p> <p>1½</p> <p>1</p> <p>½</p>
<p>SECTION-E</p> <p>(Question nos. 36 to 38 are source based/case based/passage based/integrated units of assessment questions carrying 4 marks each)</p>		

36.

The Venn diagram below represents the probabilities of three different types of Yoga, A, B and C performed by the people of a society. Further, it is given that probability of a member performing type C Yoga is 0.44.



On the basis of the above information, answer the following questions :

- (i) Find the value of x.
(ii) Find the value of y.
(iii) (a) Find $P\left(\frac{C}{B}\right)$.

OR

- (iii) (b) Find the probability that a randomly selected person of the society does Yoga of type A or B but not C.

Ans

(i) $x + 0.21 = 0.44 \Rightarrow x = 0.23$
(ii) $0.32 + y + 0.21 + 0.11 = 1 \Rightarrow y = 0.04$

(iii) (a) $P\left(\frac{C}{B}\right) = \frac{P(C \cap B)}{P(B)}$

$P(B) = 0.09 + 0.04 + 0.23 = 0.36$

$P\left(\frac{C}{B}\right) = \frac{0.23}{0.36} = \frac{23}{36}$

OR

(iii) (b) $P(\text{A or B but not C})$
 $= 0.32 + 0.09 + 0.04$
 $= 0.45$


1

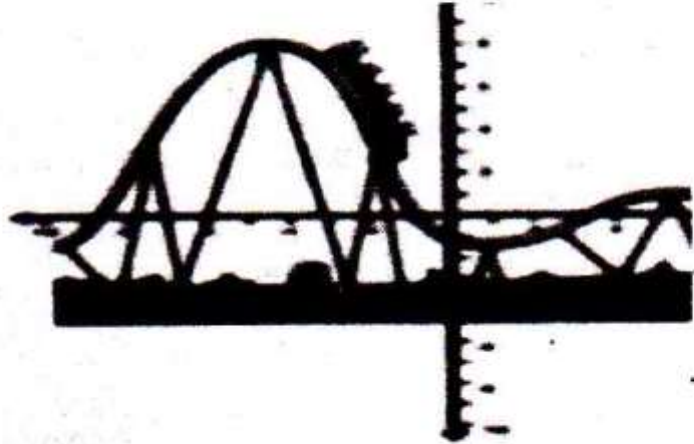
1

1

1

1 $\frac{1}{2}$
 $\frac{1}{2}$

37.	<p style="text-align: center;">Case Study - 2</p> <p>37. A tank, as shown in the figure below, formed using a combination of a cylinder and a cone, offers better drainage as compared to a flat bottomed tank.</p>  <p>A tap is connected to such a tank whose conical part is full of water. Water is dripping out from a tap at the bottom at the uniform rate of $2 \text{ cm}^3/\text{s}$. The semi-vertical angle of the conical tank is 45°.</p> <p>On the basis of given information, answer the following questions :</p> <p>(i) Find the volume of water in the tank in terms of its radius r.</p> <p>(ii) Find rate of change of radius at an instant when $r = 2\sqrt{2} \text{ cm}$.</p> <p>(iii) (a) Find the rate at which the wet surface of the conical tank is decreasing at an instant when radius $r = 2\sqrt{2} \text{ cm}$.</p> <p style="text-align: center;">OR</p> <p>(iii) (b) Find the rate of change of height 'h' at an instant when slant height is 4 cm.</p>	
Ans	<p>(i) $v = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi r^3$ [as $\theta = 45^\circ$ gives $r = h$]</p> <p>(ii) $\frac{dv}{dt} = \pi r^2 \frac{dr}{dt}$</p> <p>$\Rightarrow \left(\frac{dr}{dt} \right)_{r=2\sqrt{2}} = -\frac{1}{4\pi} \text{ cm/sec}$</p> <p>(iii)(a) $C = \pi r l = \pi r \sqrt{2} r = \sqrt{2} \pi r^2$</p> <p>$\frac{dC}{dt} = \sqrt{2} \pi 2r \frac{dr}{dt}$</p> <p>$\left(\frac{dC}{dt} \right)_{r=2\sqrt{2}} = -2 \text{ cm}^2/\text{sec}$</p> <p>OR</p> <p>(iii)(b) $l^2 = h^2 + r^2$</p> <p>$l = 4 \Rightarrow r = h = 2\sqrt{2}$</p> <p>$h = r \Rightarrow \frac{dh}{dt} = \frac{dr}{dt} = -\frac{1}{4\pi} \text{ cm/sec}$</p>	<p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>1</p>

Case Study - 3		
38.	<p>38. The equation of the path traced by a roller-coaster is given by the polynomial $f(x) = a(x + 9)(x + 1)(x - 3)$. If the roller-coaster crosses y-axis at a point $(0, -1)$, answer the following :</p>  <p>(i) Find the value of 'a'.</p> <p>(ii) Find $f''(x)$ at $x = 1$.</p>	
Ans	<p>(i) $-1 = a(-27) \Rightarrow a = \frac{1}{27}$</p> <p>(ii) $f(x) = \frac{1}{27} (x + 9)(x + 1)(x - 3)$</p> $= \frac{1}{27} (x^3 + 7x^2 - 21x - 27)$ $f'(x) = \frac{1}{27} (3x^2 + 14x - 21)$ $f''(x) = \frac{6x + 14}{27}$ $f''(1) = \frac{20}{27}$	<p>1 + 1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>