MARKING SCHEME
MATHEMATICS (Subject Code–041)
(PAPER CODE: 65/C/1)

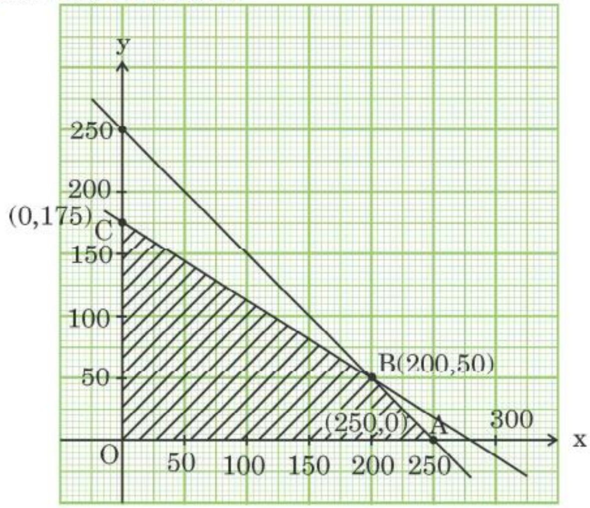
Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Marks
	SECTION A **Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each**	
1.	If A is a square matrix of order 3 and $\|A\| = 6$, then the value of $\|\text{adj } A\|$ is : (a) 6 (b) 36 (c) 27 (d) 216	
Sol.	(b) 36	**1**
2.	The value of $\int_0^{\pi/6} \sin 3x \, dx$ is : (a) $-\frac{\sqrt{3}}{2}$ (b) $-\frac{1}{3}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{3}$	
Sol.	(d) $\frac{1}{3}$	**1**
3.	If $\vec{a}$, $\vec{b}$ and $(\vec{a} + \vec{b})$ are all unit vectors and θ is the angle between $\vec{a}$ and $\vec{b}$, then the value of θ is : (a) $\frac{2\pi}{3}$ (b) $\frac{5\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$	
Sol.	(a) $\frac{2\pi}{3}$	**1**

4.	The projection of vector $\hat{i}$ on the vector $\hat{i} + \hat{j} + 2\hat{k}$ is : (a) $\frac{1}{\sqrt{6}}$ (b) $\sqrt{6}$ (c) $\frac{2}{\sqrt{6}}$ (d) $\frac{3}{\sqrt{6}}$	
Sol.	(a) $\frac{1}{\sqrt{6}}$	**1**
5.	A family has 2 children and the elder child is a girl. The probability that both children are girls is : (a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$	
Sol.	(c) $\frac{1}{2}$	**1**
6.	The vector equation of a line which passes through the point (2, – 4, 5) and is parallel to the line $\frac{x+3}{3} = \frac{4-y}{2} = \frac{z+8}{6}$ is : (a) $\vec{r} = (-2\hat{i} + 4\hat{j} - 5\hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$ (b) $\vec{r} = (2\hat{i} - 4\hat{j} + 5\hat{k}) + \lambda(3\hat{i} - 2\hat{j} + 6\hat{k})$ (c) $\vec{r} = (2\hat{i} - 4\hat{j} + 5\hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$ (d) $\vec{r} = (-2\hat{i} + 4\hat{j} - 5\hat{k}) + \lambda(3\hat{i} - 2\hat{j} - 6\hat{k})$	
Sol.	(b) $\vec{r} = (2\hat{i} - 4\hat{j} + 5\hat{k}) + \lambda(3\hat{i} - 2\hat{j} + 6\hat{k})$	**1**
7.	For which value of x, are the determinants $\begin{vmatrix} 2x & -3 \\ 5 & x \end{vmatrix}$ and $\begin{vmatrix} 10 & 1 \\ -3 & 2 \end{vmatrix}$ equal ? (a) ± 3 (b) -3 (c) ± 2 (d) 2	
Sol.	(c) ± 2	**1**
8.	The value of the cofactor of the element of second row and third column in the matrix $\begin{bmatrix} 4 & 3 & 2 \\ 2 & -1 & 0 \\ 1 & 2 & 3 \end{bmatrix}$ is : (a) 5 (b) -5 (c) -11 (d) 11	
Sol.	(b) -5	**1**

9.	The difference of the order and the degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + x^4 = 0$ is : (a) 1 (b) 2 (c) -1 (d) 0	
Sol.	(d) 0	**1**
10.	If matrix $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^2 = kA$, then the value of k is : (a) 1 (b) -2 (c) 2 (d) -1	
Sol.	(c) 2	**1**
11.	$\int \frac{\cos 2x}{\sin^2 x \cdot \cos^2 x} dx$ is equal to (a) $\tan x - \cot x + C$ (b) $-\cot x - \tan x + C$ (c) $\cot x + \tan x + C$ (d) $\tan x - \cot x - C$	
Sol.	(b) $-\cot x - \tan x + C$	**1**
12.	The integrating factor of the differential equation $(3x^2 + y)\frac{dx}{dy} = x$ is (a) $\frac{1}{x}$ (b) $\frac{1}{x^2}$ (c) $\frac{2}{x}$ (d) $-\frac{1}{x}$	
Sol.	(a) $\frac{1}{x}$	**1**
13.	The point which lies in the half-plane $2x + y - 4 \leq 0$ is : (a) (0, 8) (b) (1, 1) (c) (5, 5) (d) (2, 2)	
Sol.	(b) (1, 1)	**1**
14.	If $(\cos x)^y = (\cos y)^x$, then $\frac{dy}{dx}$ is equal to :	

	(a) $\dfrac{y \tan x + \log(\cos y)}{x \tan y - \log(\cos x)}$ (b) $\dfrac{x \tan y + \log(\cos x)}{y \tan x + \log(\cos y)}$ (c) $\dfrac{y \tan x - \log(\cos y)}{x \tan y - \log(\cos x)}$ (d) $\dfrac{y \tan x + \log(\cos y)}{x \tan y + \log(\cos x)}$	
Sol.	(d) $\dfrac{y \tan x + \log(\cos y)}{x \tan y + \log(\cos x)}$	**1**
15.	It is given that $X \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$. Then matrix X is : (a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$	
Sol.	(c) $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$	**1**
16.	If ABCD is a parallelogram and AC and BD are its diagonals, then $\vec{AC} + \vec{BD}$ is : (a) $2\vec{DA}$ (b) $2\vec{AB}$ (c) $2\vec{BC}$ (d) $2\vec{BD}$	
Sol.	(c) $2\vec{BC}$	**1**
17.	If $x = a \cos\theta + b \sin\theta$, $y = a \sin\theta - b \cos\theta$, then which one of the following is true ? (a) $y^2\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + y = 0$ (b) $y^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = 0$ (c) $y^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} - y = 0$ (d) $y^2\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - y = 0$	
Sol.	(a) $y^2\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + y = 0$	**1**

18. The corner points of the bounded feasible region of an LPP are O(0, 0), A(250, 0), B(200, 50) and C(0, 175). If the maximum value of the objective function $Z = 2ax + by$ occurs at the points A(250, 0) and B(200, 50), then the relation between a and b is :



(a) $2a = b$ (b) $2a = 3b$ (c) $a = b$ (d) $a = 2b$

Sol. (a) $2a = b$ — **1**

*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true and Reason (R) is ***not*** the correct explanation of the Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* The principal value of $\cot^{-1}(\sqrt{3})$ is $\frac{\pi}{6}$.

Reason (R) : Domain of $\cot^{-1} x$ is $\mathbb{R} - \{-1, 1\}$.

Sol. (c) Assertion (A) is true, but Reason (R) is false. — **1**

20.	*Assertion (A) :* Quadrilateral formed by vertices A(0, 0, 0), B(3, 4, 5), C(8, 8, 8) and D(5, 4, 3) is a rhombus. *Reason (R) :* ABCD is a rhombus if AB = BC = CD = DA, AC ≠ BD.	
Sol.	(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).	**1**
	SECTION B **This section comprises very short answer (VSA) type questions of 2 marks each.**	
21.	If three non-zero vectors are $\vec{a}$, $\vec{b}$ and $\vec{c}$ such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$, then show that $\vec{b} = \vec{c}$.	
Sol.	$\vec{a}.\vec{b}=\vec{a}.\vec{c} \Rightarrow \vec{a}.\left(\vec{b}-\vec{c}\right)=0$ $\Rightarrow \vec{a}=0 \text{ or } \vec{b}-\vec{c}=0 \text{ or } \vec{a} \perp \left(\vec{b}-\vec{c}\right)$ $\text{as } \vec{a} \neq 0 \Rightarrow \vec{b}-\vec{c}=0 \text{ or } \vec{a} \perp \left(\vec{b}-\vec{c}\right) \quad ...(1)$ $\text{Again, } \vec{a}\times\vec{b}=\vec{a}\times\vec{c} \Rightarrow \vec{a}\times\left(\vec{b}-\vec{c}\right)=0$ $\Rightarrow \vec{a}=0 \text{ or } \vec{b}-\vec{c}=0 \text{ or } \vec{a} \parallel \left(\vec{b}-\vec{c}\right).$ $\text{as } \vec{a} \neq 0 \Rightarrow \vec{b}-\vec{c}=0 \text{ or } \vec{a} \parallel \left(\vec{b}-\vec{c}\right) \quad ...(2)$ $\text{from } (1) \text{ and } (2), \vec{b}=\vec{c} \quad (\because \vec{a} \text{ can't be parallel and perpendicular to } \left(\vec{b}-\vec{c}\right) \text{ simultaneously.})$	**1** ½ ½
22(a).	Simplify : $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$	

Sol.	$\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)=\tan^{-1}\left(\frac{\sin\left(\frac{\pi}{2}-x\right)}{1-\cos\left(\frac{\pi}{2}-x\right)}\right)$ $=\tan^{-1}\left(\frac{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right)$ $=\tan^{-1}\left(\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)=\tan^{-1}\left(\tan\left(\frac{\pi}{2}-\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)\right)$ $=\frac{\pi}{4}+\frac{x}{2}$	½ ½ ½ ½
	OR	
22(b).	Prove that the greatest integer function $f:\mathbb{R}\rightarrow\mathbb{R}$, given by $f(x) = [x]$, is neither one-one nor onto.	
Sol.	For not one-one: $1.1, 1.2 \in R$ (domain) now, $1.1 \neq 1.2$ but $f(1.1) = f(1.2) = 1 \Rightarrow f$ is not one-one. For not onto: Let $\frac{1}{2} \in R$ (co-domain), but $[x] = \frac{1}{2}$ is not possible for x in domain. so, f is not onto.	**1** **1**
23.	Function f is defined as $f(x) = \begin{cases} 2x+2, & \text{if } x<2 \\ k, & \text{if } x=2 \\ 3x, & \text{if } x>2 \end{cases}$ Find the value of k for which the function f is continuous at x = 2.	
Sol.	As f is continuous at $x=2 \Rightarrow \lim_{x\to 2^+} f(x)=\lim_{x\to 2^-} f(x)=f(2)$ $\lim_{x\to 2^+} 3x=\lim_{x\to 2^-}(2x+2)=k$ $\Rightarrow k=6$	**1** **1**

24.	Find the intervals in which the function $f(x) = x^4 - 4x^3 + 4x^2 + 15$, is strictly increasing.	
Sol.	$f'(x) = 4x^3 - 12x^2 + 8x = 4x(x-1)(x-2)$ $f'(x) = 0$ gives $x = 0, 1, 2$ for strictly increasing, $f'(x) > 0$ $x \in (0,1) \cup (2, \infty)$	**1** **1**
25(a).	If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors such that $\|\vec{a}\| = 7$, $\|\vec{b}\| = 24$, $\|\vec{c}\| = 25$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.	
Sol.	$\vec{a} + \vec{b} + \vec{c} = \vec{0} \Rightarrow (\vec{a} + \vec{b} + \vec{c})^2 = (\vec{0})^2$ $\Rightarrow \|\vec{a}\|^2 + \|\vec{b}\|^2 + \|\vec{c}\|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$ $\Rightarrow 49 + 576 + 625 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$ $\Rightarrow \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = -625$	**1** **1**
	OR	
25(b).	If a line makes angles α, β and γ with x-axis, y-axis and z-axis respectively, then prove that $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$.	
Sol.	d.c. are $\cos\alpha, \cos\beta, \cos\gamma$ $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ $\Rightarrow (1 - \sin^2\alpha) + (1 - \sin^2\beta) + (1 - \sin^2\gamma) = 1$ $\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$	**1** **1**
	SECTION C **This section comprises of Short Answer (SA) type questions of 3 marks each.**	

26(a).	Evaluate : $$\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$$	
Sol.	$I = \int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx \quad ...(1)$ using $\int_0^a f(x) dx = \int_0^a f(a-x) dx$ $I = \int_0^{\frac{\pi}{2}} \frac{\left(\frac{\pi}{2} - x\right) \sin\left(\frac{\pi}{2} - x\right) \cos\left(\frac{\pi}{2} - x\right)}{\sin^4\left(\frac{\pi}{2} - x\right) + \cos^4\left(\frac{\pi}{2} - x\right)} dx$ $\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\left(\frac{\pi}{2} - x\right) \cos x \sin x}{\cos^4 x + \sin^4 x} dx \quad ...(2)$ adding (1) and (2) $2I = \frac{\pi}{2} \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} dx = \pi \int_0^{\frac{\pi}{4}} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} dx$ $\Rightarrow I = \frac{\pi}{2} \int_0^{\frac{\pi}{4}} \frac{\tan x \sec^2 x}{\left(\tan^2 x\right)^2 + 1} dx \quad (\because \text{dividing by } \cos^4 x)$ Putting $\tan^2 x = t$ gives $I = \frac{\pi}{4} \int_0^1 \frac{1}{t^2 + 1} dt$ $\Rightarrow I = \frac{\pi^2}{16}$	 ½ ½ 1 ½ ½
	OR	
26(b).	Evaluate : $$\int_1^3 (\|x-1\| + \|x-2\|) dx$$	

Sol.	$I=\int_1^3\left(\lvert x-1\rvert+\lvert x-2\rvert\right)dx$ $=\int_1^2\left[(x-1)-(x-2)\right]dx+\int_2^3\left[(x-1)+(x-2)\right]dx$ $=\int_1^2 1\,dx+\int_2^3(2x-3)\,dx$ $=\left[x\right]_1^2+\left[x^2-3x\right]_2^3$ $=1+2=3$	**1** ½ ½ **1**
27(a).	Find the particular solution of the differential equation $\frac{dy}{dx}=\frac{xy}{x^2+y^2}$, given that y = 1 when x = 0.	
Sol.	$\frac{dy}{dx}=\frac{xy}{x^2+y^2}\quad ...(1)$ Put $\frac{y}{x}=v$ i.e. $y=vx\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$ Equation (1) gives $v+x\frac{dv}{dx}=\frac{v}{1+v^2}$ $\Rightarrow x\frac{dv}{dx}=-\frac{v^3}{1+v^2}$ $\Rightarrow\int\frac{1+v^2}{v^3}dv=-\int\frac{dx}{x}$ $\Rightarrow\frac{-1}{2v^2}+\log\lvert v\rvert=-\log\lvert x\rvert+\log c$ putting $v=\frac{y}{x}$ and simplifying gives $-\frac{x^2}{2y^2}=\log\left\lvert\frac{c}{y}\right\rvert$ now, $x=0, y=1$ gives $c=1$ required solution is : $\frac{x^2}{2y^2}=\log\lvert y\rvert$	½ **1** ½ ½ ½
	OR	

27(b).	Find the particular solution of the differential equation $(1+x^2)\frac{dy}{dx}+2xy=\frac{1}{1+x^2}$, given that y = 0 when x = 1.	
Sol.	Given diff. eqn. can be written as $\frac{dy}{dx}+\frac{2x}{1+x^2}.y=\frac{1}{(1+x^2)^2}$ $\text{I.F.}=e^{\int\frac{2x}{1+x^2}dx}=1+x^2$ solution is given by: $y.(1+x^2)=\int\frac{1}{1+x^2}dx$ $\Rightarrow y.(1+x^2)=\tan^{-1}x+C$ Now $x=1, y=0$ gives $C=-\frac{\pi}{4}$ Reqired solution : $y.(1+x^2)=\tan^{-1}x-\frac{\pi}{4}$	½ ½ ½ ½ ½ ½
28(a).	Out of two bags, bag A contains 2 white and 3 red balls and bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag B.	
Sol.	Let E_1 : event of choosing bag A, E_2 : event of choosing bag B, A : red ball is found here, $P(E_1)=P(E_2)=\frac{1}{2}; P(A\|E_1)=\frac{3}{5}, P(A\|E_2)=\frac{5}{9}$ $P(E_2\|A)=\frac{P(E_2)P(A\|E_2)}{P(E_1)P(A\|E_1)+P(E_2)P(A\|E_2)}$ $=\frac{\frac{5}{9}\times\frac{1}{2}}{\frac{3}{5}\times\frac{1}{2}+\frac{5}{9}\times\frac{1}{2}}=\frac{25}{52}$	½ 1 1+ ½
	OR	

28(b).	Out of a group of 50 people, 20 always speak the truth. Two persons are selected at random from the group (without replacement). Find the probability distribution of number of selected persons who always speak the truth.	
Sol.	Let X be the random variable representing the number of persons who speak truth. X can takes the values 0, 1 and 2. $P(\text{speaking truth})=\frac{20}{50}, P(\text{not speaking truth})=\frac{30}{50}$ $P(X=0)=\frac{30}{50}\times\frac{29}{49}=\frac{87}{245}$ $P(X=1)=2\times\frac{20}{50}\times\frac{30}{49}=\frac{120}{245}$ $P(X=2)=\frac{20}{50}\times\frac{19}{49}=\frac{38}{245}$ Probability Distribution Table is given by: X: **0**, **1**, **2** $P(X)$: $\frac{87}{245}$, $\frac{120}{245}$, $\frac{38}{245}$	½ ½ 1½ ½
29.	Find : $$\int \frac{\cos\theta}{\sqrt{3-3\sin\theta-\cos^2\theta}}\,d\theta$$	
Sol.	$I=\int\frac{\cos\theta}{\sqrt{3-3\sin\theta-\cos^2\theta}}d\theta$ $=\int\frac{\cos\theta}{\sqrt{\sin^2\theta-3\sin\theta+2}}d\theta$ Put $\sin\theta=t\Rightarrow\cos\theta\,d\theta=dt$ $I=\int\frac{dt}{\sqrt{t^2-3t+2}}=\int\frac{dt}{\sqrt{\left(t-\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2}}$ $=\log\left\|\left(t-\frac{3}{2}\right)+\sqrt{t^2-3t+2}\right\|+C$ $=\log\left\|\left(\sin\theta-\frac{3}{2}\right)+\sqrt{\sin^2\theta-3\sin\theta+2}\right\|+C$	½ ½ ½ 1 ½

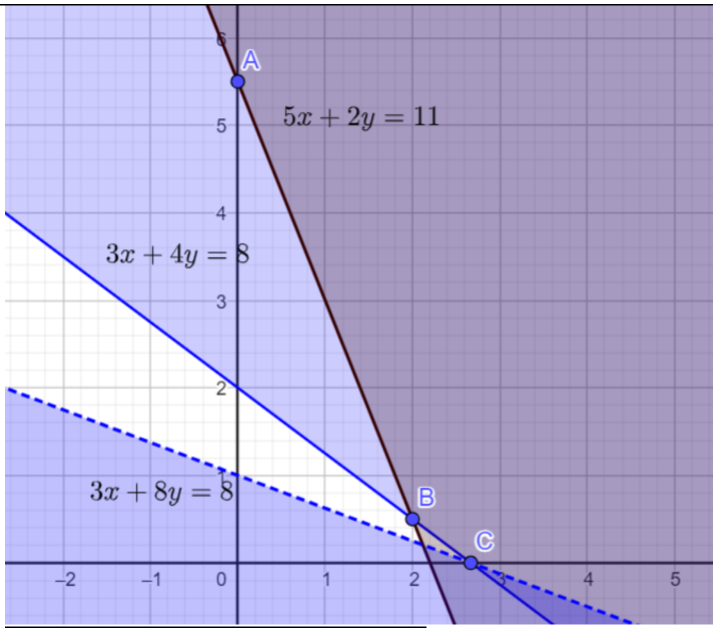
30. Solve the following Linear Programming Problem graphically :

Minimise z = 3x + 8y

subject to the constraints

$$3x + 4y \geq 8$$
$$5x + 2y \geq 11$$
$$x \geq 0,\ y \geq 0$$

Sol.



Correct graph 1 mark

Corner Point	$z = 3x + 8y$
$A\left(0, \frac{11}{2}\right)$	**44**
$B\left(2, \frac{1}{2}\right)$	**10**
$C\left(\frac{8}{3}, 0\right)$	**8**

1½

since $3x + 8y < 8$ do not have any point in common with the feasible region,

$z_{\min} = 8$ when $x = \frac{8}{3}, y = 0$ ½

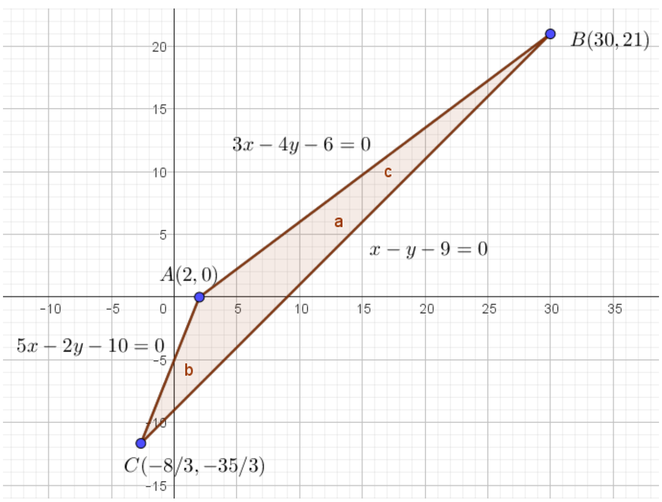
31. Find :

$$\int \frac{2x^2 + 1}{x^2(x^2 + 4)}\, dx$$

Sol.	$I=\int \frac{2x^2+1}{x^2\left(x^2+4\right)}dx$ Let $\frac{2x^2+1}{x^2\left(x^2+4\right)}=\frac{2y+1}{y\left(y+4\right)}$, where $x^2=y$ Put $\frac{2y+1}{y\left(y+4\right)}=\frac{A}{y}+\frac{B}{y+4}$ $\Rightarrow 2y+1=A\left(y+4\right)+By$ $\Rightarrow A=\frac{1}{4},B=\frac{7}{4}$ $\therefore \frac{2y+1}{y\left(y+4\right)}=\frac{1}{4y}+\frac{7}{4\left(y+4\right)}=\frac{1}{4x^2}+\frac{7}{4\left(x^2+4\right)}$ $\Rightarrow I=\frac{1}{4}\int\frac{1}{x^2}dx+\frac{7}{4}\int\frac{1}{x^2+4}dx$ $=-\frac{1}{4x}+\frac{7}{8}\tan^{-1}\left(\frac{x}{2}\right)+C$	½ 1 ½ ½ ½
	SECTION D **This section comprises of Long Answer (LA) type questions of 5 marks each.**	
32.	If matrix A = $\begin{bmatrix}3 & 2 & 1\\ 4 & 1 & 3\\ 1 & 1 & 1\end{bmatrix}$, find A^{-1} and hence solve the following system of linear equations : $3x+2y+z=2000$ $4x+y+3z=2500$ $x+y+z=900$	

Sol.	$\|A\|=3(-2)-4(1)+1(5)=-5\neq 0\Rightarrow A^{-1}$ exists. $A_{11}=-2, A_{12}=-1, A_{13}=3$ $A_{21}=-1, A_{22}=2, A_{23}=-1$ $A_{31}=5, A_{32}=-5, A_{33}=-5$ $adjA=\begin{bmatrix}-2 & -1 & 5\\ -1 & 2 & -5\\ 3 & -1 & -5\end{bmatrix}$ $A^{-1}=\frac{1}{\|A\|}adjA=-\frac{1}{5}\begin{bmatrix}-2 & -1 & 5\\ -1 & 2 & -5\\ 3 & -1 & -5\end{bmatrix}$ Given system of equations can be written as $AX=B$, where $B=\begin{bmatrix}2000\\ 2500\\ 900\end{bmatrix}$ $X=A^{-1}B$ $=-\frac{1}{5}\begin{bmatrix}-2 & -1 & 5\\ -1 & 2 & -5\\ 3 & -1 & -5\end{bmatrix}\begin{bmatrix}2000\\ 2500\\ 900\end{bmatrix}=-\frac{1}{5}\begin{bmatrix}-2000\\ -1500\\ -1000\end{bmatrix}=\begin{bmatrix}400\\ 300\\ 200\end{bmatrix}$ $\therefore x=400, y=300$ and $z=200$	**1** **1 ½** **1** ½ ½ ½
33(a).	Show that the lines $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ and $\frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}$ intersect. Also find their point of intersection.	

Sol.	line 1: $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}=\lambda$ ...(1) line 2: $\frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}=\mu$ ...(2) General points on (1) and (2) are $(3\lambda-1, 5\lambda-3, 7\lambda-5)$ and $(\mu+2, 3\mu+4, 5\mu+6)$ for the lines to intersect, $3\lambda-1=\mu+2$ ...(3) $5\lambda-3=3\mu+4$ ...(4) $7\lambda-5=5\mu+6$ ...(5) solving (3) and (4) gives $\lambda=\frac{1}{2}$ and $\mu=-\frac{3}{2}$ clearly these values of λ and μ satisfies (5) $\Rightarrow$ given lines intersect. Point of intersection is $\left(\frac{1}{2},-\frac{1}{2},-\frac{3}{2}\right)$	1 1 1 1 1
	OR	
33(b).	Find the shortest distance between the pair of lines $\frac{x-1}{2}=\frac{y+1}{3}=z$ and $\frac{x+1}{5}=\frac{y-2}{1}$; $z=2$.	

Sol.	Given lines are $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-0}{1}$ and $\frac{x+1}{5}=\frac{y-2}{1}=\frac{z-2}{0}$ In vector form, lines are $\vec{r}=(\hat{i}-\hat{j})+\lambda(2\hat{i}+3\hat{j}+\hat{k})=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=(-\hat{i}+2\hat{j}+2\hat{k})+\mu(5\hat{i}+\hat{j})=\vec{a}_2+\lambda\vec{b}_2$ now, $\vec{a}_2-\vec{a}_1=-2\hat{i}+3\hat{j}+2\hat{k}$ $\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 1\\ 5 & 1 & 0\end{vmatrix}=-\hat{i}+5\hat{j}-13\hat{k}$ $\lvert\vec{b}_1\times\vec{b}_2\rvert=\sqrt{195}$ S.D.$=\left\lvert\frac{(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)}{\lvert\vec{b}_1\times\vec{b}_2\rvert}\right\rvert$ $=\left\lvert\frac{2+15-26}{\sqrt{195}}\right\rvert=\frac{9}{\sqrt{195}}$	**1** **1** ½ **1** ½ **1**
34.	**Find the area of the triangle ABC bounded by the lines represented by the equations 5x – 2y – 10 = 0, x – y – 9 = 0 and 3x – 4y – 6 = 0, using integration method.**	
Sol.		**Correct figure 1 mark**

	solving the given equations, the vertices of triangle are $A(2,0), B(30,21)$ and $C\left(-\frac{8}{3},-\frac{35}{3}\right)$	**3**
	$\text{ar}(\triangle ABC)=\frac{3}{4}\int_{2}^{30}(x-2)\,dx-\int_{9}^{30}(x-9)\,dx+\left\|\int_{-\frac{8}{3}}^{9}(x-9)\,dx\right\|-\left\|\frac{5}{2}\int_{-\frac{8}{3}}^{2}(x-2)\,dx\right\|$ $=\frac{3}{8}(x-2)^2\Big]_2^{30}-\frac{1}{2}(x-9)^2\Big]_9^{30}+\left\|\frac{1}{2}(x-9)^2\Big]_{-\frac{8}{3}}^{9}\right\|-\left\|\frac{5}{4}(x-2)^2\Big]_{-\frac{8}{3}}^{2}\right\|$ $=294-\frac{441}{2}+\frac{1225}{18}-\frac{245}{9}=\frac{343}{3}$	**1**
35(a).	**Show that the relation S in set $\mathbb{R}$ of real numbers defined by** **$S=\{(a,b): a\le b^3,\ a\in\mathbb{R},\ b\in\mathbb{R}\}$** **is neither reflexive, nor symmetric, nor transitive.**	
Sol.	We have $S=\{(a,b: a\le b^3\}$ where $a, b\in R.$ (i) Reflexive: we observe that, $\frac{1}{2}\le\left(\frac{1}{2}\right)^3$ is not true. $\therefore\ \left(\frac{1}{2},\frac{1}{2}\right)\notin S.$ So, S is not reflexive. (ii) Symmetric: We observe that $1\le 3^3$ but $3\not\le 1^3$ i.e., (1, 3) $\in S$ but (3, 1) $\notin S$. So, S is not symmetric. (iii) Transitive: We observe that, $10\le 3^3$ and $3\le 2^3$ but $10\not\le 2^3$. i.e., (10, 3) $\in S$ and (3, 2) $\in S$ but (10, 2) $\notin S$. So, S is not transitive. $\therefore$ S is neither reflexive nor symmetric, not transitive.	**1 ½** **1 ½** **2**
	OR	

35(b).	Let R be the relation defined in the set A = {1, 2, 3, 4, 5, 6, 7} by R = {(a, b) : both a and b are either odd or even}. Show that R is an equivalence relation. Hence, find the elements of equivalence class [1].	
Sol.	$R=\{(a,b):\text{both } a \text{ and } b \text{ are either odd or even}\}$ for reflexive : Let $a\in A$ clearly both a and a are either odd or even $\therefore (a,a)\in R \Rightarrow R$ is reflexive. for symmetric : Let $a,b\in A$. Let $(a,b)\in R$ $\Rightarrow$ both a and b are either odd or even $\Rightarrow$ both b and a are either odd or even so, $(a,\mathrm{b})\in R \Rightarrow (b,a)\in R \Rightarrow R$ is symmetric. for transitive : Let $a,b,c\in A$. Let $(a,b)\in R,(b,c)\in R$ $\Rightarrow$ both a and b are either odd or even & both b and c are either odd or even $\Rightarrow$ both a and c are either odd or even so, $(a,\mathrm{b})\in R,(b,c)\in R \Rightarrow (\mathrm{a},\mathrm{c})\in R \Rightarrow R$ is transitive. equivalence class of $[1]=\{1,3,5,7\}$	**1** **1** **2** **1**
	SECTION E **This section comprises of 3 case-study based questions of 4 marks each.**	

36.	In a group activity class, there are 10 students whose ages are 16, 17, 15, 14, 19, 17, 16, 19, 16 and 15 years. One student is selected at random such that each has equal chance of being chosen and age of the student is recorded.  On the basis of the above information, answer the following questions : (i) Find the probability that the age of the selected student is a composite number. *1* (ii) Let X be the age of the selected student. What can be the value of X ? *1* (iii) (a) Find the probability distribution of random variable X and hence find the mean age. *2* **OR** (iii) (b) A student was selected at random and his age was found to be greater than 15 years. Find the probability that his age is a prime number. *2*	
Sol.	$(i)\ P(\text{age of selected student is a composite number})$ $= P(\text{age is } 14, 15 \text{ or } 16) = \frac{6}{10} = \frac{3}{5}$ $(ii)\ X$ can be $14, 15, 16, 17, 19$	**1** **1**

$(iii)(a)$

X	**14**	**15**	**16**	**17**	**19**
$P(X)$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{2}{10}$	$\frac{2}{10}$

1

$\text{mean} = \sum X.P(X)$

$= 14\left(\frac{1}{10}\right) + 15\left(\frac{2}{10}\right) + 16\left(\frac{3}{10}\right) + 17\left(\frac{2}{10}\right) + 19\left(\frac{2}{10}\right) = 16.4 \text{ years}$ **1**

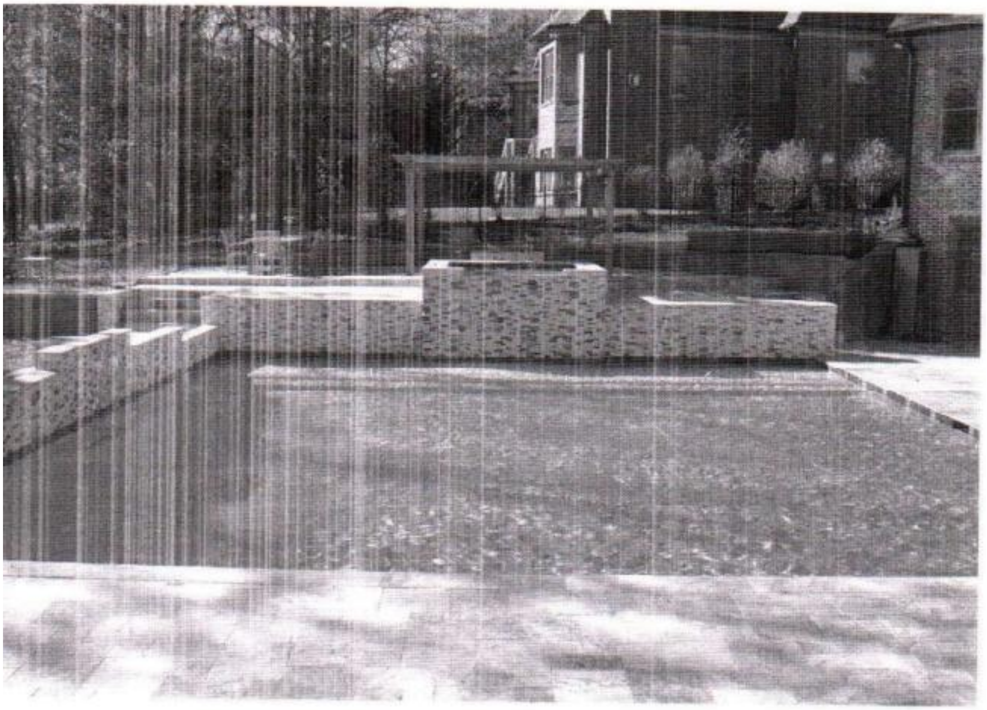
OR

$(iii)(b)\ A: \text{getting Prime number} = \{17, 19\}$

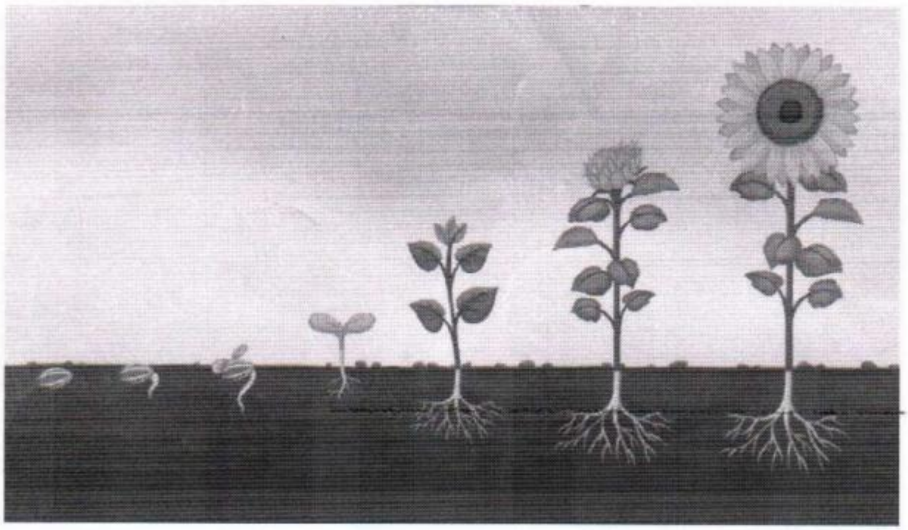
$B: \text{age is greater than 15 years} = \{16, 17, 19\}$

$A \cap B = \{17, 19\}$ **1**

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{2}{3}$ **1**

37.	A housing society wants to commission a swimming pool for its residents. For this, they have to purchase a square piece of land and dig this to such a depth that its capacity is 250 cubic metres. Cost of land is ₹ 500 per square metre. The cost of digging increases with the depth and cost for the whole pool is ₹ 4000 $(\text{depth})^2$.  Suppose the side of the square plot is x metres and depth is h metres. On the basis of the above information, answer the following questions : (i) Write cost C(h) as a function in terms of h. *1* (ii) Find critical point. *1* (iii) (a) Use second derivative test to find the value of h for which cost of constructing the pool is minimum. What is the minimum cost of construction of the pool ? *2* **OR** (iii) (b) Use first derivative test to find the depth of the pool so that cost of construction is minimum. Also, find relation between x and h for minimum cost. *2*	
Sol.		

	(i) Capacity = area×depth $= x^2h = 250 \Rightarrow x^2 = \frac{250}{h}$ C (cost) $= 500x^2 + 4000h^2$ $\Rightarrow C = 500\left(\frac{250}{h}\right) + 4000h^2 = \frac{125000}{h} + 4000h^2$	**1**
	$(ii)\ \frac{dC}{dh} = -\frac{125000}{h^2} + 8000h$ $\frac{dC}{dh} = 0 \Rightarrow h = \frac{5}{2}\,m$ or $2.5\,m$	**1**
	$(iii)(a)\ \frac{d^2C}{dh^2} = -125000\left(\frac{-2}{h^3}\right) + 8000 = \frac{250000}{h^3} + 8000$ $\left.\frac{d^2C}{dh^2}\right\|_{h=2.5m} > 0 \Rightarrow$ Cost is minimum when $h = 2.5\,m$	**1**
	Minimum cost $= C = \frac{125000}{\left(\frac{5}{2}\right)} + 4000\left(\frac{5}{2}\right)^2 = Rs.75,000$	**1**
	OR	
	$(iii)(b)$ we already have found above that $h = \frac{5}{2}\,m$ when $\frac{dC}{dh} = 0$ for the values of h less than $\frac{5}{2}$ and close to $\frac{5}{2}$, $\frac{dC}{dh} < 0$ and, for the values of h more than $\frac{5}{2}$ and close to $\frac{5}{2}$, $\frac{dC}{dh} > 0$ By first derivative test, there is a minimum at $h = \frac{5}{2}$	**1**
	Now, $x^2 = \frac{250}{h} \Rightarrow x^2 = \frac{250}{\left(\frac{5}{2}\right)} = 100 \Rightarrow x = 10\,m$	½
	also, $x = 4h$	½

38.	In an agricultural institute, scientists do experiments with varieties of seeds to grow them in different environments to produce healthy plants and get more yield. A scientist observed that a particular seed grew very fast after germination. He had recorded growth of plant since germination and he said that its growth can be defined by the function $f(x) = \frac{1}{3}x^3 - 4x^2 + 15x + 2, \ 0 \le x \le 10$ where x is the number of days the plant is exposed to sunlight.  On the basis of the above information, answer the following questions : (i) What are the critical points of the function f(x) ? *2* (ii) Using second derivative test, find the minimum value of the function. *2*	
Sol. (i) **(ii)**	$f'(x) = x^2 - 8x + 15 = (x-3)(x-5)$ $f'(x) = 0 \Rightarrow x = 3, 5$ are the critical points. Now $f''(x) = 2x - 8$ $f''(3) < 0$ and $f''(5) > 0$ so, mimimum value of $f(x)$ is at $x = 5$. min. value $= f(5) = \frac{5^3}{3} - 4(5)^2 + 15(5) + 2 = \frac{56}{3}$	**1** **1** **1** **1**