MARKING SCHEME

MATHEMATICS

SECTION A		
Q. No.	Expected Answers/Value Points	Marks
Q1	If $A = \begin{bmatrix} 1 & 4 & x \\ z & 2 & y \\ -3 & -1 & 3 \end{bmatrix}$ is a symmetric matrix, then the value of x + y + z is : (a) 10 (b) 6 (c) 8 (d) 0	
Ans	**(d) 0**	1
Q2	If A . (adj A) = $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then the value of $\|A\| + \|\text{adj } A\|$ is equal to : (a) 12 (b) 9 (c) 3 (d) 27	
Ans	**(a)12**	1
Q3	A and B are skew-symmetric matrices of same order. AB is symmetric, if : (a) AB = O (b) AB = – BA (c) AB = BA (d) BA = O	
Ans	**(c)AB = BA**	1
Q4	For what value of $x \in \left[0, \frac{\pi}{2}\right]$, is $A + A' = \sqrt{3}\, I$, where $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$? (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) 0 (d) $\frac{\pi}{2}$	

Ans	**(b)** $\frac{\pi}{6}$	1
Q5	Let A be the area of a triangle having vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3). Which of the following is correct ? (a) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm A$ (b) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm 2A$ (c) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm \frac{A}{2}$ (d) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 = A^2$	
Ans	(b) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm 2A$	1
Q6	$\int 2^{x+2} dx$ is equal to : (a) $2^{x+2} + C$ (b) $2^{x+2} \log 2 + C$ (c) $\frac{2^{x+2}}{\log 2} + C$ (d) $2 \cdot \frac{2^x}{\log 2} + C$	
Ans	(c) $\frac{2^{x+2}}{\log 2} + C$	1
Q7	$\int \frac{2 \cos 2x - 1}{1 + 2 \sin x} dx$ is equal to : (a) $x - 2 \cos x + C$ (b) $x + 2 \cos x + C$ (c) $-x - 2 \cos x + C$ (d) $-x + 2 \cos x + C$	
Ans	(b) $x + 2 \cos x + C$	1

Q8	The solution of the differential equation $\frac{dx}{x}+\frac{dy}{y}=0$ is : (a) $\frac{1}{x}+\frac{1}{y}=C$ (b) $\log x - \log y = C$ (c) $xy = C$ (d) $x + y = C$	
Ans	(c) $xy = C$	1
Q9	What is the product of the order and degree of the differential equation $\frac{d^2y}{dx^2}\sin y+\left(\frac{dy}{dx}\right)^3\cos y=\sqrt{y}$? (a) 3 (b) 2 (c) 6 (d) not defined	
Ans	**(b) 2**	1
Q10	If a vector makes an angle of $\frac{\pi}{4}$ with the positive directions of both x-axis and y-axis, then the angle which it makes with positive z-axis is : (a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{2}$ (d) 0	
Ans	(c) $\frac{\pi}{2}$	1
Q11	$\vec{a}$ and $\vec{b}$ are two non-zero vectors such that the projection of $\vec{a}$ on $\vec{b}$ is 0. The angle between $\vec{a}$ and $\vec{b}$ is : (a) $\frac{\pi}{2}$ (b) π (c) $\frac{\pi}{4}$ (d) 0	
Ans	(a) $\frac{\pi}{2}$	1
Q12	In Δ ABC, $\vec{AB}=\hat{i}+\hat{j}+2\hat{k}$ and $\vec{AC}=3\hat{i}-\hat{j}+4\hat{k}$. If D is mid-point of BC, then vector $\vec{AD}$ is equal to : (a) $4\hat{i}+6\hat{k}$ (b) $2\hat{i}-2\hat{j}+2\hat{k}$ (c) $\hat{i}-\hat{j}+\hat{k}$ (d) $2\hat{i}+3\hat{k}$	
Ans	(d) $2\hat{i}+3\hat{k}$	1

Q13	The value of λ for which the angle between the lines $\vec{r} = \hat{i} + \hat{j} + \hat{k} + p(2\hat{i} + \hat{j} + 2\hat{k})$ and $\vec{r} = (1+q)\hat{i} + (1+q\lambda)\hat{j} + (1+q)\hat{k}$ is $\frac{\pi}{2}$ is : (a) -4 (b) 4 (c) 2 (d) -2	
Ans	(a) -4	1
Q14	If $P(A \cap B) = \frac{1}{8}$ and $P(\bar{A}) = \frac{3}{4}$, then $P\left(\frac{B}{A}\right)$ is equal to : (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{6}$ (d) $\frac{2}{3}$	
Ans	(a) $\frac{1}{2}$	1
Q15	The value of k for which function $f(x) = \begin{cases} x^2, & x \geq 0 \\ kx, & x < 0 \end{cases}$ is differentiable at $x = 0$ is : (a) 1 (b) 2 (c) any real number (d) 0	
Ans	**(d) 0**	1
Q16	If $y = \frac{\cos x - \sin x}{\cos x + \sin x}$, then $\frac{dy}{dx}$ is : (a) $-\sec^2\left(\frac{\pi}{4} - x\right)$ (b) $\sec^2\left(\frac{\pi}{4} - x\right)$ (c) $\log\left\|\sec\left(\frac{\pi}{4} - x\right)\right\|$ (d) $-\log\left\|\sec\left(\frac{\pi}{4} - x\right)\right\|$	
Ans	(a) $-\sec^2\left(\frac{\pi}{4} - x\right)$	1

Q17

The number of feasible solutions of the linear programming problem given as

Maximize $z = 15x + 30y$ subject to constraints :

$3x + y \leq 12,\ x + 2y \leq 10,\ x \geq 0, y \geq 0$ is

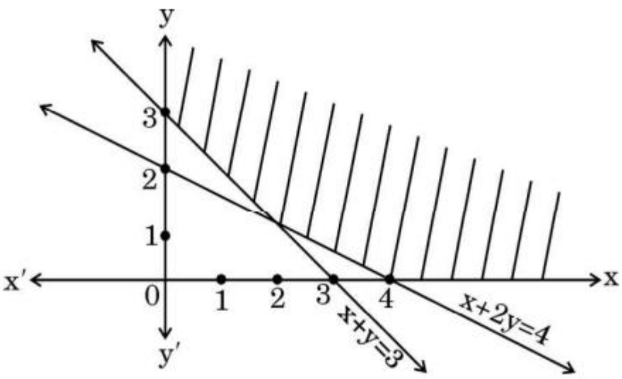
(a) 1 (b) 2

(c) 3 (d) infinite

Ans	**(d) infinite**	1

Q18

The feasible region of a linear programming problem is shown in the figure below :



Which of the following are the possible constraints ?

(a) $x + 2y \geq 4,\ x + y \leq 3,\ x \geq 0,\ y \geq 0$

(b) $x + 2y \leq 4,\ x + y \leq 3,\ x \geq 0,\ y \geq 0$

(c) $x + 2y \geq 4,\ x + y \geq 3,\ x \geq 0,\ y \geq 0$

(d) $x + 2y \geq 4,\ x + y \geq 3,\ x \leq 0,\ y \leq 0$

Ans	(c) $x + 2y \geq 4,\ x + y \geq 3,\ x \geq 0,\ y \geq 0$	1

*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

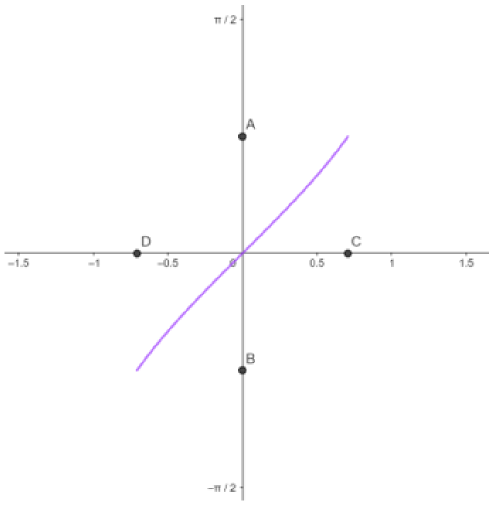
Q19

Assertion (A) : Range of $\left[\sin^{-1} x + 2\cos^{-1} x\right]$ is $[0, \pi]$.

Reason (R) : Principal value branch of $\sin^{-1} x$ has range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Ans	(d) Assertion (A) is false and Reason (R) is true.	1

Q20	Assertion (A): A line through the points (4, 7, 8) and (2, 3, 4) is parallel to a line through the points (−1, −2, 1) and (1, 2, 5). Reason (R): Lines $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu \vec{b_2}$ are parallel if $\vec{b_1} \cdot \vec{b_2} = 0$.	
Ans	(c) Assertion (A) is true and Reason (R) is false.	1
	SECTION B	
Q21	If $\vec{r} = 3\hat{i} - 2\hat{j} + 6\hat{k}$, find the value of $(\vec{r} \times \hat{j}).(\vec{r} \times \hat{k}) - 12$.	
Ans	$(\vec{r} \times \hat{j}).(\vec{r} \times \hat{k}) - 12 = (3\hat{k} - 6\hat{i}).(-3\hat{j} - 2\hat{i}) - 12$ $= 12 - 12 = 0$	$1\frac{1}{2}$ ½
Q22	If the angle between the lines $\frac{x-5}{\alpha} = \frac{y+2}{-5} = \frac{z+\frac{24}{5}}{\beta}$ and $\frac{x}{1} = \frac{y}{0} = \frac{z}{1}$ is $\frac{\pi}{4}$, find the relation between α and β.	
Ans	$\cos\frac{\pi}{4} = \frac{\lvert\alpha . 1 + 0 + \beta\rvert}{\sqrt{\alpha^2 + \beta^2 + 25}\sqrt{2}} \Rightarrow \frac{1}{\sqrt{2}} = \frac{\lvert\alpha + \beta\rvert}{\sqrt{\alpha^2 + \beta^2 + 25}\sqrt{2}}$ Squaring both sides, we get $\alpha^2 + \beta^2 + 2\alpha\beta = \alpha^2 + \beta^2 + 25$ $\Rightarrow \alpha\beta = \frac{25}{2}$	1 ½ ½
Q23	If f(x) = a(tan x − cot x), where a > 0, then find whether f(x) is increasing or decreasing function in its domain.	
Ans	$f'(x) = a(\sec^2 x + \text{cosec}^2 x)$ As a > 0 and $\sec^2 x$, $\text{cosec}^2 x$ are squares, $f'(x) > 0$ ∴ f(x) is an increasing function in its domain.	1 ½ ½
Q24(a)	Evaluate : $3\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + 2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}(0)$	
Ans	(a) Given expression $= \frac{3\pi}{4} + \frac{2\pi}{6} + \frac{\pi}{2}$ $= \frac{19\pi}{12}$	1½ ½

	OR	
Q24(b)	Draw the graph of $f(x) = \sin^{-1} x,\ x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$. Also, write range of f(x).	
Ans	(b)  Correct graph Here, the points A, B, C and D are respectively $\left(0, \frac{\pi}{4}\right), \left(0, -\frac{\pi}{4}\right), \left(\frac{1}{\sqrt{2}}, 0\right), \left(-\frac{1}{\sqrt{2}}, 0\right)$. Range = $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$	1 1
Q25(a)	If $y = x^{\frac{1}{x}}$, then find $\frac{dy}{dx}$ at $x = 1$.	
Ans	(a) $y = x^{1/x}$ $\Rightarrow \log y = \frac{1}{x} \log x$ $\Rightarrow \frac{1}{y}\frac{dy}{dx} = -\frac{\log x}{x^2} + \frac{1}{x^2} \Rightarrow \frac{dy}{dx} = x^{\frac{1}{x}} \frac{(1 - \log x)}{x^2}$ $\Rightarrow \left(\frac{dy}{dx}\right)_{x=1} = 1$	 ½ 1 ½
	OR	
Q25(b)	If $x = a \sin 2t,\ y = a(\cos 2t + \log \tan t)$, then find $\frac{dy}{dx}$.	
Ans	(b) $\frac{dx}{dt} = 2a \cos 2t$ $\frac{dy}{dt} = 2a\left(-\sin 2t + \frac{\sec^2 t}{2 \tan t}\right) = 2a \frac{\cos^2 2t}{\sin 2t}$ $\frac{dy}{dx} = \cot 2t$	½ 1 ½

	SECTION C	
Q26(a)	Find the general solution of the differential equation : $\frac{d}{dx}\left(xy^2\right) = 2y\left(1+x^2\right)$	
Ans	(a) Given differential equation is $2xy\frac{dy}{dx} + y^2 = 2y(1 + x^2)$ $\Rightarrow \frac{dy}{dx} + \frac{y}{2x} = \frac{1}{x} + x$ Integrating factor = $e^{\int \frac{1}{2x}dx} = e^{\log \sqrt{x}} = \sqrt{x}$ Solution is given by $y\sqrt{x} = \int \left(\frac{1}{\sqrt{x}} + x^{\frac{3}{2}}\right)dx$ $\Rightarrow y\sqrt{x} = 2\sqrt{x} + \frac{2x^{\frac{5}{2}}}{5} + C$,or $y = 2 + \frac{2x^2}{5} + \frac{C}{\sqrt{x}}$	 ½ 1 1 ½
	OR	
Q26(b)	Solve the following differential equation : $xe^{\frac{y}{x}} - y + x\frac{dy}{dx} = 0$	
Ans	(b) Given differential equation is $\frac{dy}{dx} = \frac{y}{x} - e^{\frac{y}{x}}$ Let y = vx $\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ The given equation becomes $v + x\frac{dv}{dx} = v - e^{v}$ $\Rightarrow - e^{-v}dv = \frac{dx}{x}$ Integrating both sides, we get $e^{-v} = \log\lvert x\rvert + C$ $\Rightarrow e^{-\frac{y}{x}} = \log\lvert x\rvert + C$	½ ½ ½ 1 ½
Q27	Evaluate: $\int_1^3 \frac{\sqrt{4-x}}{\sqrt{x}+\sqrt{4-x}}dx$	
Ans	Let $I = \int_1^3 \frac{\sqrt{4-x}}{\sqrt{x}+\sqrt{4-x}}dx$ $I = \int_1^3 \frac{\sqrt{4-(4-x)}}{\sqrt{4-x}+\sqrt{4-(4-x)}}dx$ $2I = \int_1^3 \frac{\sqrt{4-x}+\sqrt{x}}{\sqrt{4-x}+\sqrt{x}}dx = \int_1^3 1dx$ $2I = x\rvert_1^3 = 2$ $\Rightarrow I = 1$	 1 1 ½ ½

Q28	Evaluate : $\int_1^e \frac{1}{\sqrt{4x^2-(x\log x)^2}}dx$	
Ans	Let $I = \int_1^e \frac{1}{\sqrt{4x^2-(x\log x)^2}}dx$ $= \int_1^e \frac{1}{x\sqrt{4-(\log x)^2}}dx$ [Let $\log x = t \Rightarrow \frac{1}{x}dx = dt$] $= \int_0^1 \frac{dt}{\sqrt{4-t^2}}$ $= \sin^{-1}\frac{t}{2}\Big\|_0^1 = \frac{\pi}{6}$	1 ½ ½ 1
Q29(a)	Find: $\int \frac{\cos x}{\sin 3x}dx$	
Ans	(a)$I = \int \frac{\cos x}{3\sin x - 4\sin^3 x}dx$ Let $\sin x = t \Rightarrow \cos x\, dx = dt$ $I = \int \frac{dt}{3t-4t^3}$ $= \int \frac{1}{t^3\left(\frac{3}{t^2}-4\right)}dt$ Let $\frac{3}{t^2} - 4 = z \Rightarrow -\frac{6}{t^3}dt = dz$ $I = -\frac{1}{6}\int \frac{dz}{z}$ $= -\frac{1}{6}\log\|z\| + C$ $= -\frac{1}{6}\log\|3\,\mathrm{cosec}^2 x - 4\| + C$	½ ½ ½ ½ ½ ½
	OR	
Q29(b)	Find: $\int x^2\log(x^2+1)dx$	
Ans	(b)Let $I = \int x^2\log(x^2+1)dx$ $= \log(x^2+1)\cdot\frac{x^3}{3} - \int \frac{2x}{x^2+1}\cdot\frac{x^3}{3}dx$ $= \frac{x^3}{3}\log(x^2+1) - \frac{2}{3}\int \frac{x^4}{x^2+1}dx$	1 ½

$= \frac{x^3}{3}\log(x^2+1) - \frac{2}{3}\int\left(x^2 - 1 + \frac{1}{x^2+1}\right)dx$ ½

$= \frac{x^3}{3}\log(x^2+1) - \frac{2}{3}\left[\frac{x^3}{3} - x + \tan^{-1}x\right] + C$ 1

Q30 Determine graphically the minimum value of the following objective function :

$z = 500x + 400y$

subject to constraints

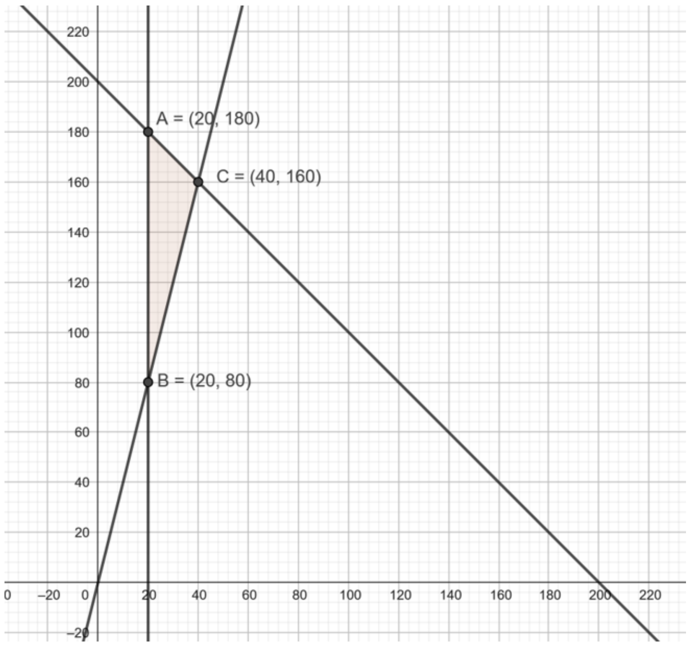
$x + y \le 200,$

$x \ge 20,$

$y \ge 4x,$

$y \ge 0.$

Ans



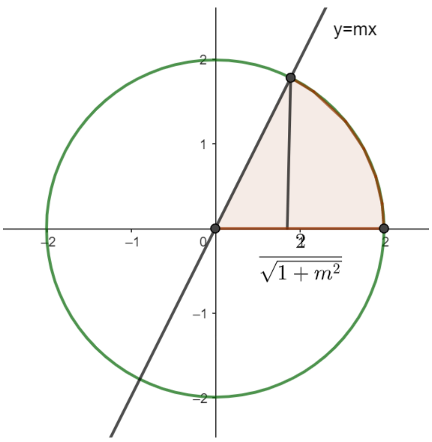
2 for correct graph

Corner points	Value of Z = 500x + 400y
(20, 180)	82000
(40, 160)	84000
(20, 80)	42000 → Minimum

1

Q31(a) (a) A pair of dice is thrown simultaneously. If X denotes the absolute difference of numbers obtained on the pair of dice, then find the probability distribution of X.

Ans	(a) X: 0, 1, 2, 3, 4, 5 P(X): $\frac{6}{36}$, $\frac{10}{36}$, $\frac{8}{36}$, $\frac{6}{36}$, $\frac{4}{36}$, $\frac{2}{36}$	1 ½ 1 ½
	OR	
Q31(b)	There are two coins. One of them is a biased coin such that P (head) : P (tail) is 1 : 3 and the other coin is a fair coin. A coin is selected at random and tossed once. If the coin showed head, then find the probability that it is a biased coin.	
Ans	(b)E_1 = Biased coin is selected $\Rightarrow P(E_1)=\frac{1}{2}$ E_2 = Fair coin is selected $\Rightarrow P(E_2)=\frac{1}{2}$ A = Head appeared on tossing a selected coin . $P\left(\frac{A}{E_1}\right)=\frac{1}{4}$, $P\left(\frac{A}{E_2}\right)=\frac{1}{2}$ By Bayes' Theorem $P\left(\frac{E_1}{A}\right)=\frac{P(E_1)\,P\left(\frac{A}{E_1}\right)}{P(E_1)\,P\left(\frac{A}{E_1}\right)+P(E_2)\,P\left(\frac{A}{E_2}\right)}$ $=\frac{\frac{1}{2}\cdot\frac{1}{4}}{\frac{1}{2}\cdot\frac{1}{4}+\frac{1}{2}\cdot\frac{1}{2}}$ $=\frac{1}{3}$	½ 1 1 ½
	SECTION D	
Q32	Show that a function $f:\mathbb{R}\to\mathbb{R}$ defined as $f(x)=\frac{5x-3}{4}$ is both one-one and onto.	
Ans	Let $x_1, x_2 \in R$ (Domain) such that $f(x_1) = f(x_2)$ $\Rightarrow \frac{5x_1-3}{4}=\frac{5x_2-3}{4}$ $\Rightarrow 5x_1 = 5x_2 \Rightarrow x_1 = x_2$ $\therefore$ f is one-one.	$2\frac{1}{2}$

	Let $y \in R$ (co-domain). Then f(x) = y for some x. if, $y = \frac{5x-3}{4}, i.e., if, \ x = \frac{4y+3}{5}$, which $\in R$(Domain) Thus, for every y$\in R$(co − domain), there exists $\frac{4y+3}{5} \in R$(domain) such that $f\left(\frac{4y+3}{5}\right) = y$ $\therefore$ Range of f = R = codomain of f. Hence, f is onto.	$2\frac{1}{2}$
Q33	The area of the region bounded by the line y = mx (m > 0), the curve $x^2 + y^2 = 4$ and the x-axis in the first quadrant is $\frac{\pi}{2}$ units. Using integration, find the value of m.	
Ans	 $x^2 + y^2 = 4$ and y = mx $\Rightarrow x^2 + m^2x^2 = 4 \Rightarrow x = \frac{2}{\sqrt{1+m^2}}$ x- coordinate of the required point of intersection is $\frac{2}{\sqrt{1+m^2}}$. According to question, $\int_0^{\frac{2}{\sqrt{1+m^2}}} mx\, dx + \int_{\frac{2}{\sqrt{1+m^2}}}^{2} \sqrt{4-x^2}dx = \frac{\pi}{2}$ $\Rightarrow m \left.\frac{x^2}{2}\right\|_0^{\frac{2}{\sqrt{1+m^2}}} + \left.\frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\frac{x}{2}\right\|_{\frac{2}{\sqrt{1+m^2}}}^{2} = \frac{\pi}{2}$ $\Rightarrow \frac{2m}{1+m^2} + \pi - \frac{2m}{1+m^2} - 2\sin^{-1}\frac{1}{\sqrt{1+m^2}} = \frac{\pi}{2}$	1 for correct figure 1 1+1 ½

	$\Rightarrow \frac{\pi}{4} = \sin^{-1}\frac{1}{\sqrt{1+m^2}}$ $\Rightarrow \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1+m^2}} \Rightarrow m^2 + 1 = 2$ $\Rightarrow m = 1$ (as $m > 0$)	½
Q34(a)	If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, then show that $A^3 - 6A^2 + 7A + 2I = 0$	
Ans	(a) getting, $A^2 = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$	$1\frac{1}{2}$
	getting, $A^3 = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$	$1\frac{1}{2}$
	$\therefore\ A^3 - 6A^2 + 7A + 2I =$	
	$\begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$	1
	$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$	1
	OR	
Q34(b)	If $A = \begin{bmatrix} 3 & 2 \\ 5 & -7 \end{bmatrix}$, then find A^{-1} and use it to solve the following system of equations : $3x + 5y = 11,\ 2x - 7y = -3.$	
Ans	(b)adj A = $\begin{bmatrix} -7 & -2 \\ -5 & 3 \end{bmatrix}$	1
	$\lvert A \rvert = -31$	1
	$A^{-1} = \frac{-1}{31}\begin{bmatrix} -7 & -2 \\ -5 & 3 \end{bmatrix}$	½
	Given system of equations is $\begin{bmatrix} 3 & 5 \\ 2 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 11 \\ -3 \end{bmatrix}$	
	which is A′X = B, where $X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 11 \\ -3 \end{bmatrix}$	½

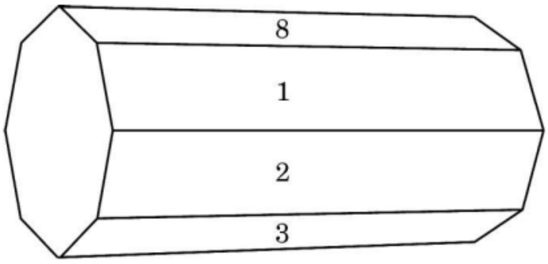
	$\Rightarrow X = (A')^{-1}B$ $\Rightarrow X = (A^{-1})'B$ $= \frac{-1}{31}\begin{bmatrix} -7 & -5 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 11 \\ -3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $\therefore x = 2, \; y = 1$	½ ½ 1
Q35(a)	Find the value of b so that the lines $\frac{x-1}{2} = \frac{y-b}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ are intersecting lines. Also, find the point of intersection of these given lines.	
Ans	(a)As lines are intersecting, $(\overrightarrow{a_2} - \overrightarrow{a_1}).((\overrightarrow{b_1} \times \overrightarrow{b_2}) = 0$ $\Rightarrow \begin{vmatrix} 3 & 1-b & -3 \\ 2 & 3 & 4 \\ 5 & 2 & 1 \end{vmatrix} = 0$ $\Rightarrow b = 2$ Any point on the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ is $(2\lambda + 1, 3\lambda + 2, 4\lambda + 3), \; \lambda \in R$ For the point of intersection, this point must lie on the line $\frac{x-4}{5} = \frac{y-1}{2} = z$ $\Rightarrow \frac{2\lambda + 1 - 4}{5} = \frac{3\lambda + 2 - 1}{2} = 4\lambda + 3$ $\Rightarrow \lambda = -1$ $\therefore$ point of intersection is $(-1, -1, -1)$	1 1 1 1 ½ ½
	OR	
Q35(b)	Find the equations of all the sides of the parallelogram ABCD whose vertices are A(4, 7, 8), B(2, 3, 4), C(– 1, – 2, 1) and D(1, 2, 5). Also, find the coordinates of the foot of the perpendicular from A to CD.	

Ans	(b)Equation of the line AB : $\frac{x-4}{2} = \frac{y-7}{4} = \frac{z-8}{4}$	½
	Equation of the line BC : $\frac{x-2}{3} = \frac{y-3}{5} = \frac{z-4}{3}$	½
	Equation of the line CD : $\frac{x+1}{1} = \frac{y+2}{2} = \frac{z-1}{2}$	½
	Equation of the line DA : $\frac{x-4}{3} = \frac{y-7}{5} = \frac{z-8}{3}$	½
	Let P be foot of perpendicular from A to CD.	
	$\therefore$ Coordinates of P are $(\lambda - 1, 2\lambda - 2, 2\lambda + 1)$ for some λ	1
	d.r.'s of AP are $(\lambda - 5, 2\lambda - 9, 2\lambda - 7)$	½
	since AP⊥CD	
	$\Rightarrow 1(\lambda - 5) + 2(2\lambda - 9) + 2(2\lambda - 7) = 0$	½
	$\Rightarrow 9\lambda = 37 \quad \Rightarrow \lambda = \frac{37}{9}$	½
	$\therefore$ Coordinates of P are $\left(\frac{28}{9}, \frac{56}{9}, \frac{83}{9}\right)$	½

SECTION E

Q36

An octagonal prism is a three-dimensional polyhedron bounded by two octagonal bases and eight rectangular side faces. It has 24 edges and 16 vertices.



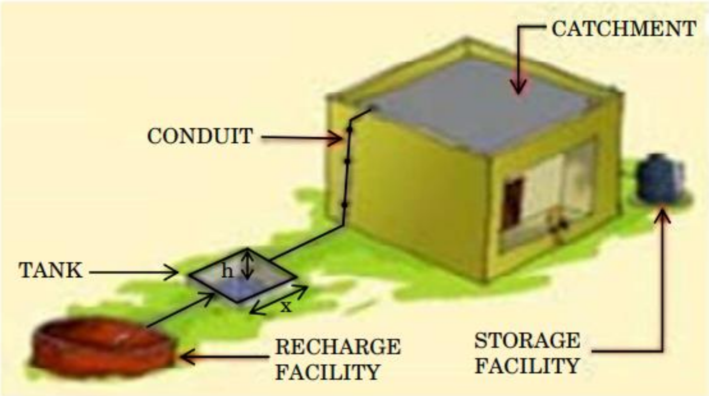
The prism is rolled along the rectangular faces and number on the bottom face (touching the ground) is noted. Let X denote the number obtained on the bottom face and the following table give the probability distribution of X.

X :	1	2	3	4	5	6	7	8
P(X) :	p	2p	2p	p	2p	p^2	$2p^2$	$7p^2 + p$

	Based on the above information, answer the following questions : (i) Find the value of p. (ii) Find $P(X > 6)$. (iii) (a) Find $P(X = 3m)$, where m is a natural number. **OR** (iii) (b) Find the mean $E(X)$.	
Ans(i)	(i) $10p^2 + 9p = 1$ $\Rightarrow p = \frac{1}{10}$	½ ½
Ans(ii)	(ii) $P(X > 6) = 9p^2 + p$ $= \frac{9}{100} + \frac{1}{10}$ $= \frac{19}{100}$	½ ½
Ans(iii)	(iii)(a) $P(X = 3\text{ m}) = P(3) + P(6)$ $\Rightarrow 2p + p^2 = \frac{21}{100}$	1 1
	OR	
Ans(iii)	(iii)(b) $E(X) = \sum XP(X) = p + 4p + 6p + 4p + 10p + 6p^2 + 14p^2 + 56p^2 + 8p$ $= 33p + 76p^2$ $= \frac{406}{100}$ or $\frac{203}{50}$	1 ½ ½

Q37 In order to set up a rain water harvesting system, a tank to collect rain water is to be dug. The tank should have a square base and a capacity of 250 m^3. The cost of land is ₹ 5,000 per square metre and cost of digging increases with depth and for the whole tank, it is ₹ 40,000 h^2, where h is the depth of the tank in metres. x is the side of the square base of the tank in metres.

ELEMENTS OF A TYPICAL RAIN WATER HARVESTING SYSTEM



Based on the above information, answer the following questions :

(i) Find the total cost C of digging the tank in terms of x.

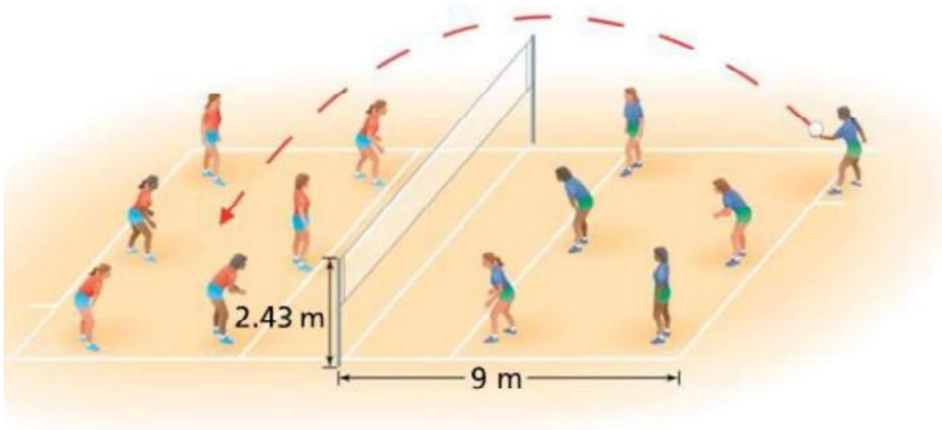
(ii) Find $\frac{dC}{dx}$.

(iii) (a) Find the value of x for which cost C is minimum.

OR

(iii) (b) Check whether the cost function C(x) expressed in terms of x is increasing or not, where x > 0.

Ans(i)	(i) $C = 40000h^2 + 5000x^2$ as $x^2h = 250$ $\Rightarrow C = \frac{40000\,(250)^2}{x^4} + 5000x^2$	 ½ ½
Ans(ii)	(ii) $\frac{dC}{dx} = \frac{-160000\,(250)^2}{x^5} + 10000x$	1
Ans(iii)	(iii)(a) For minimum cost $\frac{dC}{dx} = 0$ $\Rightarrow 10000x^6 = 250 \times 250 \times 160000$ $\Rightarrow x = 10$ showing $\frac{d^2C}{dx^2} > 0$ at x = 10 ∴ cost is minimum when x = 10	½ 1 ½
	OR	

Ans(iii)	(iii)(b)$\frac{dC}{dx} = \frac{-160000\,(250)^2}{x^4} + 10000x$ $\frac{dC}{dx} = 0$ gives x = 10 $\frac{dC}{dx} > 0$ in $(10, \infty)$ and $\frac{dC}{dx} < 0$ in (0, 10). Hence, cost function is neither increasing nor decreasing for x > 0	½ 1 ½
Q38	A volleyball player serves the ball which takes a parabolic path given by the equation $h(t) = -\frac{7}{2}t^2 + \frac{13}{2}t + 1$, where h(t) is the height of ball at any time t (in seconds), $(t \geq 0)$.  Based on the above information, answer the following questions : (i) Is h(t) a continuous function ? Justify. (ii) Find the time at which the height of the ball is maximum.	
Ans(i)	(i)$h(t) = -\frac{7}{2}t^2 + \frac{13}{2}t + 1$ Clearly h(t) is a polynomial function, hence continuous. Hence h(t) is a continuous function.	2
Ans(ii)	(ii)For maximum height , $\frac{dh}{dt} = 0 \Rightarrow -7t + \frac{13}{2} = 0$ $\Rightarrow t = \frac{13}{14}$ $\frac{d^2h}{dt^2} = -7 < 0$ $\therefore$ height is maximum at $t = \frac{13}{14}$	1 ½ ½