MARKING SCHEME
MATHEMATICS (Subject Code–041)
(PAPER CODE: 65/5/1)

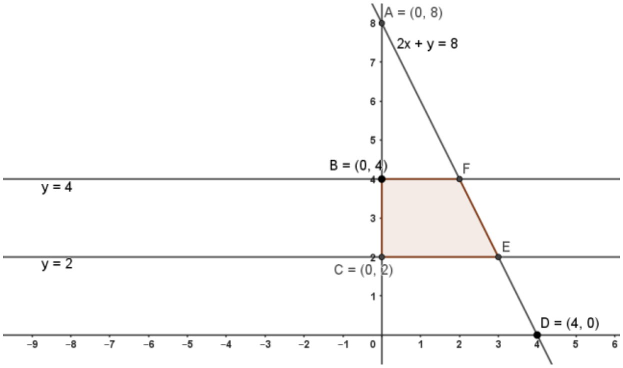
Q. No.	**EXPECTED OUTCOMES/VALUE POINTS**	**Marks**
	SECTION A **Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each**	
1.	Let A = {3, 5}. Then number of reflexive relations on A is (a) 2 (b) 4 (c) 0 (d) 8	
Sol.	(b) 4	**1**
2.	$\sin\left[\frac{\pi}{3}+\sin^{-1}\left(\frac{1}{2}\right)\right]$ is equal to (a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$	
Sol.	(a) 1	**1**
3.	If for a square matrix A, $A^2 - A + I = O$, then A^{-1} equals (a) A (b) A + I (c) I – A (d) A – I	
Sol.	(c) I – A	**1**
4.	If $A = \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}$, $B = \begin{bmatrix}x & 0\\1 & 1\end{bmatrix}$ and $A = B^2$, then x equals (a) ± 1 (b) –1 (c) 1 (d) 2	
Sol.	(c) 1	**1**

5.	If $\begin{vmatrix} \alpha & 3 & 4 \\ 1 & 2 & 1 \\ 1 & 4 & 1 \end{vmatrix} = 0$, then the value of α is (a) 1 (b) 2 (c) 3 (d) 4	
Sol.	(d) 4	**1**
6.	The derivative of x^{2x} w.r.t. x is (a) x^{2x-1} (b) $2x^{2x} \log x$ (c) $2x^{2x}(1 + \log x)$ (d) $2x^{2x}(1 - \log x)$	
Sol.	(c) $2x^{2x}(1 + \log x)$	**1**
7.	The function $f(x) = [x]$, where $[x]$ denotes the greatest integer less than or equal to x, is continuous at (a) $x = 1$ (b) $x = 1.5$ (c) $x = -2$ (d) $x = 4$	
Sol.	(b) $x = 1{\cdot}5$	**1**
8.	If $x = A \cos 4t + B \sin 4t$, then $\dfrac{d^2x}{dt^2}$ is equal to (a) x (b) $-x$ (c) $16x$ (d) $-16x$	
Sol.	(d) $\boldsymbol{-16\,x}$	**1**
9.	The interval in which the function $f(x) = 2x^3 + 9x^2 + 12x - 1$ is decreasing, is (a) $(-1, \infty)$ (b) $(-2, -1)$ (c) $(-\infty, -2)$ (d) $[-1, 1]$	
Sol.	(b) $(\boldsymbol{-2, -1})$	**1**
10.	$\displaystyle\int \frac{\sec x}{\sec x - \tan x}\, dx$ equals (a) $\sec x - \tan x + c$ (b) $\sec x + \tan x + c$ (c) $\tan x - \sec x + c$ (d) $-(\sec x + \tan x) + c$	
Sol.	(b) $\boldsymbol{\sec x + \tan x + c}$	**1**

11.	$\int_{-1}^{1} \frac{\lvert x-2 \rvert}{x-2}\, dx, x \neq 2$ का मान है : (a) 1 (b) –1 (c) 2 (d) –2	
Sol.	(d) $-\mathbf{2}$	**1**
12.	The sum of the order and the degree of the differential equation $\frac{d}{dx}\left(\left(\frac{dy}{dx}\right)^3\right)$ is (a) 2 (b) 3 (c) 5 (d) 0	
Sol.	Due to error in the question, 1 mark should be awarded to each student who attempted the question	**1**
13.	Two vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are collinear if (a) $a_1b_1 + a_2b_2 + a_3b_3 = 0$ (b) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$ (c) $a_1 = b_1, a_2 = b_2, a_3 = b_3$ (d) $a_1 + a_2 + a_3 = b_1 + b_2 + b_3$	
Sol.	(b) $\boldsymbol{\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}}$	**1**
14.	The magnitude of the vector $6\hat{i} - 2\hat{j} + 3\hat{k}$ is (a) 1 (b) 5 (c) 7 (d) 12	
Sol.	(c) 7	**1**
15.	If a line makes angles of 90°, 135° and 45° with the x, y and z axes respectively, then its direction cosines are (a) $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ (b) $-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$ (c) $\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}$ (d) $0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$	
Sol.	(a) $\mathbf{0}, -\frac{\mathbf{1}}{\sqrt{\mathbf{2}}}, \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}$	**1**

16.	The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is (a) 0° (b) 30° (c) 45° (d) 90°	
Sol.	(d) **90°**	**1**
17.	If for any two events A and B, $P(A) = \frac{4}{5}$ and $P(A \cap B) = \frac{7}{10}$, then P(B/A) is equal to (a) $\frac{1}{10}$ (b) $\frac{1}{8}$ (c) $\frac{7}{8}$ (d) $\frac{17}{20}$	
Sol.	(c) $\mathbf{\frac{7}{8}}$	**1**
18.	Five fair coins are tossed simultaneously. The probability of the events that atleast one head comes up is (a) $\frac{27}{32}$ (b) $\frac{5}{32}$ (c) $\frac{31}{32}$ (d) $\frac{1}{32}$	
Sol.	(c) $\mathbf{\frac{31}{32}}$	**1**
	Assertion – Reason Based Questions In the following questions **19** and **20**, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices : (a) Both (A) and (R) are true and (R) is the correct explanation of (A). (b) Both (A) and (R) are true, but (R) is not the correct explanation of (A). (c) (A) is true and (R) is false. (d) (A) is false, but (R) is true.	
19.	**Assertion (A) :** Two coins are tossed simultaneously. The probability of getting two heads, if it is known that at least one head comes up, is $\frac{1}{3}$. **Reason (R) :** Let E and F be two events with a random experiment, then $P(F/E) = \frac{P(E \cap F)}{P(E)}$.	

Sol.	(a) Both (A) and (R) are true and (R) is the correct explanation of (A)	**1**
20.	**Assertion (A) :** $\int_{2}^{8} \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}\, dx = 3$ **Reason (R) :** $\int_{a}^{b} f(x)\, dx = \int_{a}^{b} f(a+b-x)\, dx$	
Sol.	(a) Both (A) and (R) are true and (R) is the correct explanation of (A)	**1**
	SECTION B **This section comprises very short answer (VSA) type questions of 2 marks each.**	
21.	Write the domain and range (principle value branch) of the following functions : $f(x) = \tan^{-1} x$	
Sol.	Domain = R ; Range = $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	**1+1**
22(a).	If $f(x) = \begin{cases} x^2 , & \text{if } x \geq 1 \\ x , & \text{if } x < 1 \end{cases}$, then show that f is not differentiable at $x = 1$.	
Sol.	Here $\left.\begin{array}{l} \text{RHD} = \lim_{h \to 0} \frac{f(1+h)-f(1)}{h} = 2 \\ \text{LHD} = \lim_{h \to 0} \left[\frac{f(1-h)-f(1)}{-h}\right] = 1 \end{array}\right\}$ Since RHD $\neq$ LHD $\therefore$ f is not differentiable at x = 1.	$\mathbf{1\frac{1}{2}}$ $\mathbf{\frac{1}{2}}$
22(b).	Find the value(s) of 'λ', if the function $f(x) = \begin{cases} \frac{\sin^2 \lambda x}{x^2} , & \text{if } x \neq 0 \text{ is continuous at } x = 0. \\ 1 , & \text{if } x = 0 \end{cases}$	

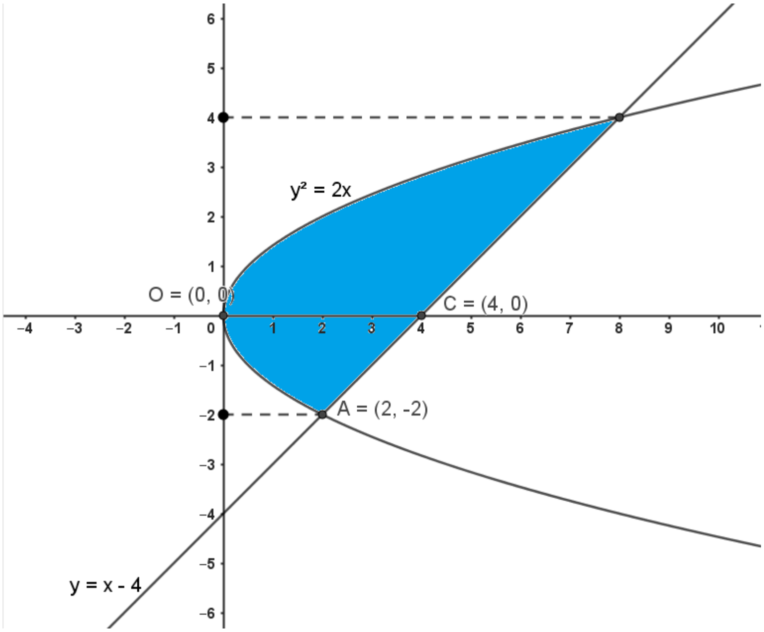
Sol.	$\lim_{x \to 0} f(x) = \lim_{x \to 0} \left(\frac{\sin^2 \lambda x}{x^2}\right) = \lim_{x \to 0} \left[\frac{\sin^2 \lambda x}{(\lambda x)^2} \cdot \lambda^2\right] = \lambda^2$ Since f(x) is continuous at x = 0 $\lim_{x \to 0} f(x) = f(0)$ $\Rightarrow \lambda^2 = 1 \Rightarrow \lambda = \pm 1$	**1** $\frac{1}{2}$ $\frac{1}{2}$
23.	Sketch the region bounded by the lines $2x + y = 8$, $y = 2$, $y = 4$ and the y-axis. Hence, obtain its area using integration.	
Sol.	 Required area = $\int_2^4 \frac{1}{2}\ (8 - y)dy$ $= \frac{1}{2} \left\|8y - \frac{y^2}{2}\right\|_2^4$ = 5	**$\frac{1}{2}$ for correct figure** $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
24(a).	If the vectors $\vec{a}$ and $\vec{b}$ are such that $\|\vec{a}\| = 3$, $\|\vec{b}\| = \frac{2}{3}$ and $\vec{a} \times \vec{b}$ is a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$.	

Sol.	Let θ be the angle between $\vec{a}$ and $\vec{b}$ Since $\vec{a} \times \vec{b}$ is a unit vector, we have $\left\|\vec{a} \times \vec{b}\right\| = 1$ $\Rightarrow \|\vec{a}\|\ \left\|\vec{b}\right\| \sin\theta = 1$ $\Rightarrow \sin\theta = \frac{1}{2}$, or θ = 30° (or $\frac{\pi}{6}$)	**1** **1**
24(b).	Find the area of a parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.	
Sol.	Here $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix} = 20\hat{i} + 5\hat{j} - 5\hat{k}$ $\Rightarrow \left\|\vec{a} \times \vec{b}\right\| = \sqrt{400 + 25 + 25} = \sqrt{450}$ **Area of parallelogram =** $\left\|\vec{a} \times \vec{b}\right\| = \sqrt{450} = 15\sqrt{2}$	$1\frac{1}{2}$ $\frac{1}{2}$
25.	Find the vector and the cartesian equations of a line that passes through the point A(1, 2, –1) and parallel to the line $5x - 25 = 14 - 7y = 35z$.	
Sol.	The given line is $\frac{x-5}{\frac{1}{5}} = \frac{y-2}{-\frac{1}{7}} = \frac{z}{\frac{1}{35}}$, or $\frac{x-5}{7} = \frac{y-2}{-5} = \frac{z}{1}$ So, the required vector equation of the line passing through (1,2,-1) is $\vec{r} = (\hat{\imath} + 2\hat{\jmath} - \hat{k}) + \lambda(7\hat{\imath} - 5\hat{\jmath} + \hat{k})$	**1** $\frac{1}{2}$

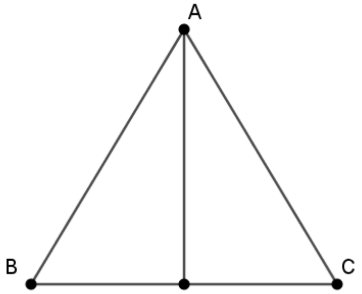
	Cartesian equation of the line is $\frac{x-1}{7}=\frac{y-2}{-5}=\frac{z+1}{1}$	$\frac{\mathbf{1}}{\mathbf{2}}$
	SECTION C **This section comprises of Short Answer (SA) type questions of 3 marks each.**	
26.	If A = $\begin{bmatrix}1&2&3\\3&-2&1\\4&2&1\end{bmatrix}$, then show that $A^3 - 23A - 40I = O$.	
Sol.	Getting $A^2 = \begin{bmatrix}19&4&8\\1&12&8\\14&6&15\end{bmatrix}$ Getting $A^3 = \begin{bmatrix}63&46&69\\69&-6&23\\92&46&63\end{bmatrix}$ $A^3 - 23A - 40I =$ $\begin{bmatrix}63&46&69\\69&-6&23\\92&46&63\end{bmatrix}-\begin{bmatrix}23&46&69\\69&-46&23\\92&46&23\end{bmatrix}-\begin{bmatrix}40&0&0\\0&40&0\\0&0&40\end{bmatrix}$ $= \begin{bmatrix}\mathbf{0}&\mathbf{0}&\mathbf{0}\\\mathbf{0}&\mathbf{0}&\mathbf{0}\\\mathbf{0}&\mathbf{0}&\mathbf{0}\end{bmatrix} = \mathbf{O}$	**1** **1** $\frac{\mathbf{1}}{\mathbf{2}}$ $\frac{\mathbf{1}}{\mathbf{2}}$
27(a).	Differentiate $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ w.r.t. $\sin^{-1}(2x\sqrt{1-x^2})$.	
Sol.	Let x = sin θ. Then $U = \sec^{-1}\left(\frac{1}{\sqrt{1-\sin^2\theta}}\right) = \sec^{-1}\left(\frac{1}{\cos\theta}\right)$	

	$= \sec^{-1}(\sec\theta) = \theta = sin^{-1}x$ $\Rightarrow \frac{dU}{dx} = \frac{1}{\sqrt{1-x^2}}$ and V = $\sin^{-1}\{2\sin\theta\sqrt{1-\sin^2\theta}\}$ $= \sin^{-1}[2\sin\theta\cos\theta] = 2\theta = 2\,sin^{-1}x$ $\Rightarrow \frac{dV}{dx} = \frac{2}{\sqrt{1-x^2}}$ $\Rightarrow \frac{dU}{dV} = \frac{dU/dx}{dV/dx} = \frac{1}{2}$ **Note: If the substitution is made as $x = cos\,\theta$, answer will be $-\frac{1}{2}$**	**1** **1** **1**
27(b).	If y = tan x + sec x, then prove that $\frac{d^2y}{dx^2} = \frac{\cos x}{(1-\sin x)^2}$.	
Sol.	y = tan x + sec x = $\frac{\sin x + 1}{\cos x}$ $\Rightarrow \frac{dy}{dx} = \frac{cos\ x\,(\cos x) + (\sin x + 1)\sin x}{cos^2\ x}$ $= \frac{\cos^2 x + \sin^2 x + \sin x}{\cos^2 x} = \frac{1+\sin x}{1-\sin^2 x} = \frac{1}{1-\sin x}$ $\Rightarrow \frac{d^2y}{dx^2} = \frac{(1-\sin x).0 - 1(0-\cos x)}{(1-\sin x)^2} = \frac{\cos x}{(1-\sin x)^2}$	$\mathbf{1\frac{1}{2}}$ $\mathbf{1\frac{1}{2}}$
28(a).	Evaluate : $\int_0^{2\pi} \frac{1}{1+e^{\sin x}}\,dx$	

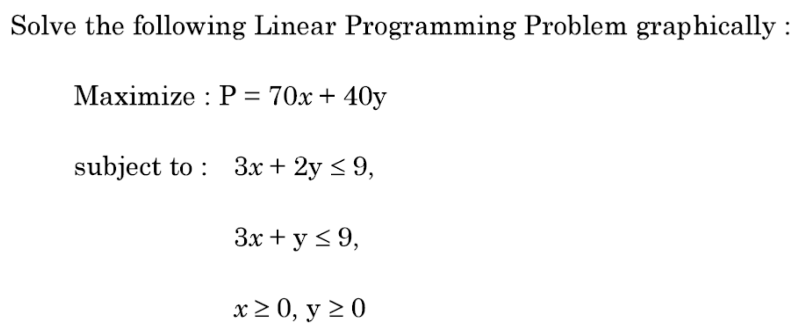
Sol.	Let I = $\int_0^{2\pi} \frac{1}{1+e^{\sin x}}\,dx = \int_0^{2\pi} \frac{1}{1+e^{\sin(2\pi - x)}}\,dx$ $= \int_0^{2\pi} \frac{1}{1+e^{-\sin x}}dx = \int_0^{2\pi} \frac{e^{\sin x}}{e^{\sin x}+1}dx$ $\Rightarrow 2I = \int_0^{2\pi} \frac{e^{\sin x}+1}{e^{\sin x}+1}\,dx = \int_0^{2\pi} 1\,.\,dx = 2\pi$ $\Rightarrow$ **I = π**	**1** **1** $\frac{1}{2}$ $\frac{1}{2}$
28(b).	Find : $\int \frac{x^4}{(x-1)\,(x^2+1)}\,dx$	
Sol.	$I = \int \frac{x^4}{(x-1)(x^2+1)}dx = \int \left[x+1+\frac{1}{(x-1)(x^2+1)}\right]dx$ $= \frac{x^2}{2} + x + \int \left[\frac{1}{2(x-1)} - \frac{1}{2}\,\frac{(x+1)}{(x^2+1)}\right]dx$ (Using partial fractions) $= \frac{x^2}{2} + \mathbf{x} + \frac{1}{2}\,\mathbf{log}\,\lvert \mathbf{x-1} \rvert - \frac{1}{4}\,\mathbf{log}\,\lvert \mathbf{x^2+1} \rvert - \frac{1}{2}\,\mathbf{tan^{-1}\,x + C}$	**1** $\frac{1}{2}$**+1** $\frac{1}{2}$
29.	Find the area of the following region using integration : $\{(x, y) : y^2 \leq 2x \text{ and } y \geq x - 4\}$	

Sol.	 Solving $y^2 = 2x$ and $y = x - 4$, we get $y = 4$ or -2 Required area = $\int_{-2}^{4}\left[(y + 4) - \frac{y^2}{2}\right]dy$ $= \left\|\frac{y^2}{2} + 4y - \frac{1}{6}y^3\right\|_{-2}^{4}$ **= 18**	**1 mark for correct figure** $\frac{1}{2}$ **1** $\frac{1}{2}$
30(a).	Find the coordinates of the foot of the perpendicular drawn from the point P(0, 2, 3) to the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$.	
Sol.	General point on the given line is M $(5\lambda - 3, 2\lambda + 1, 3\lambda - 4)$ Direction ratios of PM are $5\lambda - 3, 2\lambda - 1, 3\lambda - 7$	**1**

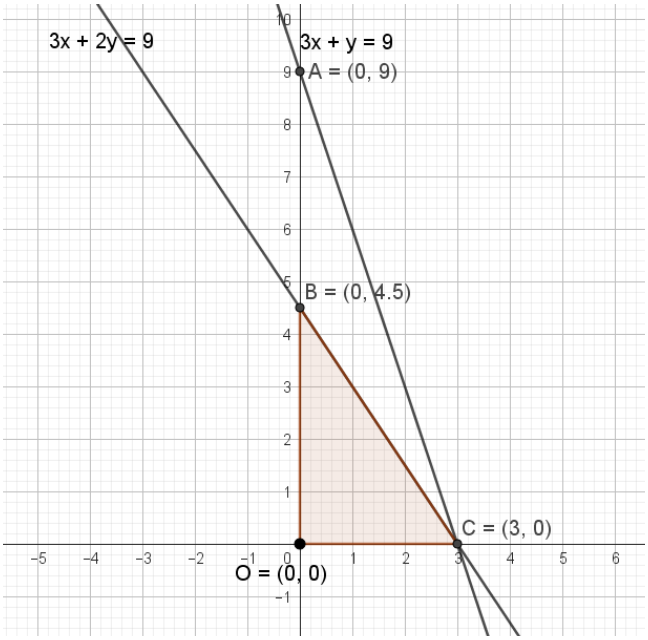
	If this point is the foot of the perpendicular from the point P (0, 2, 3), then PM is perpendicular to the line. Thus, $(5\lambda - 3).5 + (2\lambda - 1).2 + (3\lambda - 7).3 = 0$ $\Rightarrow \lambda = 1$ **Hence co-ordinates of M are (2, 3, – 1)**	**1** **1**
30(b).	Three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ satisfy the condition $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Evaluate the quantity $\mu = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$, if $\lvert \vec{a} \rvert = 3$, $\lvert \vec{b} \rvert = 4$ and $\lvert \vec{c} \rvert = 2$.	
Sol.	$(\vec{a} + \vec{b} + \vec{c})^2 = 0$ $\Rightarrow \vec{a}^2 + \vec{b}^2 + \vec{c}^2 + 2(\mu) = 0$ $\Rightarrow \mu = -\frac{29}{2}$	**1** **1** **1**
31.	Find the distance between the lines : $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$; $\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$	
Sol.	Here $\vec{a_1} = \hat{i} + 2\hat{j} - 4\hat{k}$, $\vec{b_1} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ $\vec{a_2} = 3\hat{i} + 3\hat{j} - 5\hat{k}$, $\vec{b_2} = 4\hat{i} + 6\hat{j} + 12\hat{k}$ Here, $\vec{b_1}$ and $\vec{b_2}$ are parallel vectors.	$\frac{1}{2}$

	$\overrightarrow{a_2}-\overrightarrow{a_1}=2\hat{\imath}+\hat{\jmath}-\hat{k}, \quad \vec{b}=2\hat{\imath}+3\hat{\jmath}+6\hat{k}$ Thus, $(\overrightarrow{a_2}-\overrightarrow{a_1})\times\vec{b}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 2 & 1 & -1\\ 2 & 3 & 6\end{vmatrix}=9\hat{i}-14\hat{j}+4\hat{k}$ Distance between the lines = $\left\lvert\frac{(\overrightarrow{a_2}-\overrightarrow{a_1})\times\vec{b}}{\lvert\vec{b}\rvert}\right\rvert$ $=\frac{\sqrt{81+196+16}}{\sqrt{4+9+36}}$ $=\frac{\sqrt{293}}{7}$ units.	$\frac{1}{2}$ 1 1
	SECTION D **This section comprises of Long Answer (LA) type questions of 5 marks each.**	
32(a).	The median of an equilateral triangle is increasing at the rate of $2\sqrt{3}$ cm/s. Find the rate at which its side is increasing.	
Sol.	In an equilateral triangle, median is same as altitude. Let 'h' denote the length of the median (or altitude) and 'x' be the side of Δ ABC.  Then, $h=\frac{\sqrt{3}}{2}x$ or $x=\frac{2h}{\sqrt{3}}$ ________ (i)	1 2

	It is given that $\frac{dh}{dt} = 2\sqrt{3}$ So, by (i) we have $\frac{dx}{dt} = \frac{2}{\sqrt{3}} \frac{dh}{dt} \Rightarrow \frac{dx}{dt} = 4$ **Thus, the side of Δ ABC is increasing at the rate of 4 cm/sec.**	**1** **1**
32(b).	Sum of two numbers is 5. If the sum of the cubes of these numbers is least, then find the sum of the squares of these numbers.	
Sol.	Let the two numbers be x and y. Then, x + y = 5 or y = 5 – x Let S denote the sum of the cubes of these numbers. Then $S = x^3 + y^3 = x^3 + (5 - x)^3$ $\frac{dS}{dx} = 3x^2 - 3(5 - x)^2 = 15(2x - 5)$ Now $\frac{dS}{dx}$ = 0, gives $x = \frac{5}{2}$ Showing S is minimum at $x = \frac{5}{2}$ So, the two numbers are $\frac{5}{2}$ and $\frac{5}{2}$ $\Rightarrow \boldsymbol{x^2 + y^2 = \frac{25}{4} + \frac{25}{4} = \frac{25}{2}}$	$\boldsymbol{\frac{1}{2}}$ **1** **1** $\boldsymbol{\frac{1}{2}}$ **1** **1**
33.	Evaluate : $\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1} (\sin x)\, dx$	
Sol.	Let I = $\int_0^{\pi/2} \sin 2x \tan^{-1} (\sin x)\, dx$.	

	$= \int_0^{\pi/2} 2 \sin x \cos x \tan^{-1} (\sin x)\, dx$ Put sin x = t so that cos x dx = dt Thus, $I = 2\int_0^1 t \tan^{-1} t\, dt$ $= 2\left[\left\|\frac{t^2}{2} \tan^{-1} t\right\|_0^1 - \int_0^1 \frac{1}{1+t^2} \cdot \frac{t^2}{2}\, dt\right]$ $= 2 \cdot \frac{1}{2} \cdot \frac{\pi}{4} - \int_0^1 \frac{t^2}{1+t^2}\, dt$ $= \frac{\pi}{4} - \int_0^1 \left[1 - \frac{1}{1+t^2}\right] dt$ $= \frac{\pi}{4} - \| t \|_0^1 + \left\|\tan^{-1} t\right\|_0^1$ $= \frac{\pi}{4} - 1 + \frac{\pi}{4}$ $= \frac{\pi}{2} - 1$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 1 1 1 $\frac{1}{2}$
34.	 Solve the following Linear Programming Problem graphically : Maximize : $P = 70x + 40y$ subject to : $3x + 2y \le 9$, $3x + y \le 9$, $x \ge 0, y \ge 0$	

Sol.



3 for correct figure and shading

Corner Points	Value of P
O (0,0)	0
B (0,4.5)	180
C (3,0)	210 → Max Value

$1\frac{1}{2}$

Maximum value of P =210 at $x = 3$ and y=0

$\frac{1}{2}$

35(a).	In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{3}$. What is the probability that the student knows the answer, given that he answered it correctly ?	
Sol.	Let events A, B and E be defined as: A : Student knows the answer B : Student guesses the answer E : student answered correctly	**1**
	$P(A) = \frac{3}{5}$, $P(B) = \frac{2}{5}$	$\frac{\mathbf{1}}{\mathbf{2}}$
	Here, $P\left(\frac{E}{A}\right) = 1$ and $P\left(\frac{E}{B}\right) = \frac{1}{3}$	**1**
	By Bayes' Theorem $P\left(\frac{A}{E}\right) = \dfrac{P(A)\,.\,P\left(\frac{E}{A}\right)}{P(A)\,.\,P\left(\frac{E}{A}\right) + P(B)\,.\,P\left(\frac{E}{B}\right)}$	

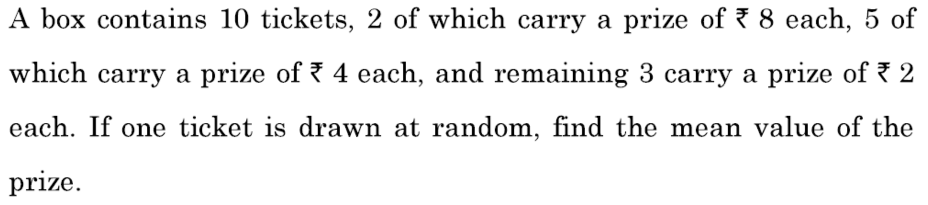
$$= \frac{\frac{3}{5}\times 1}{\left(\frac{3}{5}\times 1\right)+\left(\frac{2}{5}\times\frac{1}{3}\right)}$$

$1\frac{1}{2}$

$$= \frac{9}{11}$$

1

35(b).



A box contains 10 tickets, 2 of which carry a prize of ₹ 8 each, 5 of which carry a prize of ₹ 4 each, and remaining 3 carry a prize of ₹ 2 each. If one ticket is drawn at random, find the mean value of the prize.

Sol.

Let X denote the prize value.

Here X can take values of 8, 4 and 2.

1

$P(X = 8) = \frac{2}{10}$, or $\frac{1}{5}$

$P(X = 4) = \frac{5}{10}$, or $\frac{1}{2}$

$P(X = 2) = \frac{3}{10}$

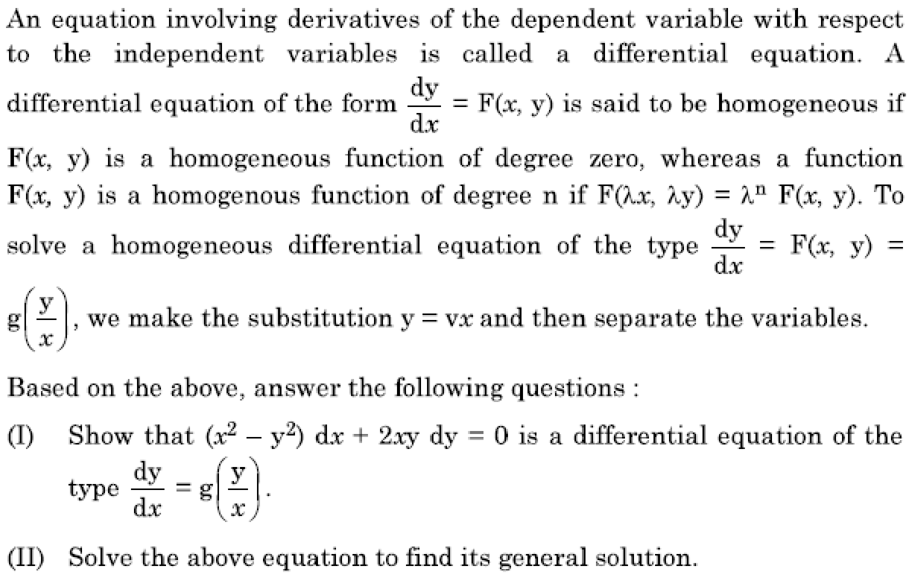
X	8	4	2
P(X)	$\frac{1}{5}$	$\frac{1}{2}$	$\frac{3}{10}$
XP(X)	$\frac{8}{5}$	$\frac{4}{2}$	$\frac{6}{10}$

3

Hence, Mean value of X = $\sum X\ P(X) = \frac{8}{5} + 2 + \frac{6}{10}$

	$= \frac{42}{10}$ or ₹ 4.20	**1**
	SECTION E **This section comprises of 3 case-study based questions of 4 marks each.**	
36.	An organization conducted bike race under two different categories – Boys and Girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project. Let B = $\{b_1, b_2, b_3\}$ and G = $\{g_1, g_2\}$, where B represents the set of Boys selected and G the set of Girls selected for the final race.  Based on the above information, answer the following questions : (I) How many relations are possible from B to G ? (II) Among all the possible relations from B to G, how many functions can be formed from B to G ? (III) Let R : B → B be defined by R = $\{(x, y) : x$ and y are students of the same sex$\}$. Check if R is an equivalence relation. **OR** (III) A function f : B → G be defined by f = $\{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$. Check if f is bijective. Justify your answer.	
Sol.	(I) Number of relations = $2^6 = 64$ (II) Number of possible functions = $2^3 = 8$ (III) R is an equivalence relation as it is reflexive, symmetric and transitive	**1** **1** **2**

	OR Since f is not one-one function $\therefore f$ is not bijective	**1** **1**
37.	Gautam buys 5 pens, 3 bags and 1 instrument box and pays a sum of ₹ 160. From the same shop, Vikram buys 2 pens, 1 bag and 3 instrument boxes and pays a sum of ₹ 190. Also Ankur buys 1 pen, 2 bags and 4 instrument boxes and pays a sum of ₹ 250. Based on the above information, answer the following questions : (I) Convert the given above situation into a matrix equation of the form AX = B. (II) Find $\lvert A \rvert$. (III) Find A^{-1}. **OR** (III) Determine $P = A^2 - 5A$.	
Sol.	(I) Matrix equation is AX = B, where $A = \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \ B = \begin{bmatrix} 160 \\ 190 \\ 250 \end{bmatrix}$ where x is the number of pens bought, y the number of bags and z the number of instrument boxes. (II) $\lvert A \rvert = 5(4 - 6) - 3(8 - 3) + 1(4 - 1) = -22$ (III) adj (A) = $\begin{bmatrix} -2 & -5 & 3 \\ -10 & 19 & -7 \\ 8 & -13 & -1 \end{bmatrix}' = \begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}$	**1** **1** **1**

	$\Rightarrow A^{-1} = \frac{1}{(-22)}\begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}$	**1**
	OR	
	$P = A^2 - 5A = \begin{bmatrix} 32 & 20 & 18 \\ 15 & 13 & 17 \\ 13 & 13 & 23 \end{bmatrix} - \begin{bmatrix} 25 & 15 & 5 \\ 10 & 5 & 15 \\ 5 & 10 & 20 \end{bmatrix}$	$\mathbf{1+\frac{1}{2}}$
	$= \begin{bmatrix} 7 & 5 & 13 \\ 5 & 8 & 2 \\ 8 & 3 & 3 \end{bmatrix}$	$\mathbf{\frac{1}{2}}$
38.	 An equation involving derivatives of the dependent variable with respect to the independent variables is called a differential equation. A differential equation of the form $\frac{dy}{dx} = F(x, y)$ is said to be homogeneous if F(x, y) is a homogeneous function of degree zero, whereas a function F(x, y) is a homogenous function of degree n if $F(\lambda x, \lambda y) = \lambda^n F(x, y)$. To solve a homogeneous differential equation of the type $\frac{dy}{dx} = F(x, y) = g\left(\frac{y}{x}\right)$, we make the substitution y = vx and then separate the variables. Based on the above, answer the following questions : (I) Show that $(x^2 - y^2)\,dx + 2xy\,dy = 0$ is a differential equation of the type $\frac{dy}{dx} = g\left(\frac{y}{x}\right)$. (II) Solve the above equation to find its general solution.	
Sol.	(I) $(x^2 - y^2)dx + 2xydy = 0$ $\Rightarrow \frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$	**1**

	$= \frac{\left(\frac{y}{x}\right)^2 - 1}{2\left(\frac{y}{x}\right)}$ $= g\left(\frac{y}{x}\right)$ (II) $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dy}{dx}$ $v + x\frac{dv}{dx} = \frac{v^2 - 1}{2v} - v = \frac{-1 - v^2}{2v}$ $\Rightarrow \int \frac{2v}{1+v^2}\,dv = -\int \frac{dx}{x}$ $\Rightarrow \log \lvert 1 + v^2 \rvert + \log \lvert x \rvert = \log C$ or $x\left(1 + \frac{y^2}{x^2}\right) = C$ or $x^2 + y^2 = Cx$	**1** $\mathbf{\frac{1}{2}}$ **1** $\mathbf{\frac{1}{2}}$