# MARKING SCHEME

## MATHEMATICS (Subject Code–041)

## (PAPER CODE: 65/1/1)

### Section A

| Q.No. | EXPECTED OUTCOMES/VALUE POINTS | Marks |
|---|---|---|
| | **SECTION A**<br>Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of **1 mark each.** | |
| 1. | A function $f : R_+ \rightarrow R$ (where $R_+$ is the set of all non-negative real numbers) defined by $f(x) = 4x + 3$ is :<br>(A) one-one but not onto<br>(B) onto but not one-one<br>(C) both one-one and onto<br>(D) neither one-one nor onto | |
| Sol. | (A) one-one but not onto | 1 |
| 2. | If a matrix has 36 elements, the number of possible orders it can have, is :<br>(A) 13 (B) 3<br>(C) 5 (D) 9 | |
| Sol. | (D) 9 | 1 |
| 3. | Which of the following statements is true for the function $f(x) = \begin{cases} x^2 + 3, & x \neq 0 \\ 1, & x = 0 \end{cases}$ ?<br>(A) f(x) is continuous and differentiable $\forall x \in \mathbb{R}$<br>(B) f(x) is continuous $\forall x \in \mathbb{R}$<br>(C) f(x) is continuous and differentiable $\forall x \in \mathbb{R} - \{0\}$<br>(D) f(x) is discontinuous at infinitely many points | |
| Sol. | (C) f(x) is continuous and differentiable $\forall x \in \boldsymbol{R} - \{0\}$ | 1 |

| | | |
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| 4. | Let f(x) be a continuous function on [a, b] and differentiable on (a, b). Then, this function f(x) is strictly increasing in (a, b) if<br>(A) $f'(x) < 0, \forall x \in (a, b)$<br>(B) $f'(x) > 0, \forall x \in (a, b)$<br>(C) $f'(x) = 0, \forall x \in (a, b)$<br>(D) $f(x) > 0, \forall x \in (a, b)$ | |
| Sol. | (B) $f'(x) > 0, \forall x \in (a, b)$ | 1 |
| 5. | If $\begin{bmatrix} x+y & 2 \\ 5 & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, then the value of $\left(\frac{24}{x} + \frac{24}{y}\right)$ is :<br>(A) 7 (B) 6<br>(C) 8 (D) 18 | |
| Sol. | (D) 18 | 1 |
| 6. | $\int_a^b f(x)\,dx$ is equal to :<br>(A) $\int_a^b f(a-x)\,dx$ (B) $\int_a^b f(a+b-x)\,dx$<br>(C) $\int_a^b f(x-(a+b))\,dx$ (D) $\int_a^b f((a-x)+(b-x))\,dx$ | |
| Sol. | (B) $\int_a^b f(a+b-x)dx$ | 1 |
| 7. | Let θ be the angle between two unit vectors $\hat{a}$ and $\hat{b}$ such that $\sin\theta = \frac{3}{5}$<br>Then, $\hat{a} \cdot \hat{b}$ is equal to :<br>(A) $\pm\frac{3}{5}$ (B) $\pm\frac{3}{4}$<br>(C) $\pm\frac{4}{5}$ (D) $\pm\frac{4}{3}$ | |
| Sol. | (C) $\pm\frac{4}{5}$ | 1 |
| 8. | The integrating factor of the differential equation $(1-x^2)\frac{dy}{dx} + xy = ax$,<br>$-1 < x < 1$, is :<br>(A) $\frac{1}{x^2-1}$ (B) $\frac{1}{\sqrt{x^2-1}}$<br>(C) $\frac{1}{1-x^2}$ (D) $\frac{1}{\sqrt{1-x^2}}$ | |
| Sol. | (D) $\frac{1}{\sqrt{1-x^2}}$ | 1 |

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| 9. | If the direction cosines of a line are $\sqrt{3}$ k, $\sqrt{3}$ k, $\sqrt{3}$ k, then the value of k is :<br>(A) $\pm 1$ (B) $\pm \sqrt{3}$<br>(C) $\pm 3$ (D) $\pm \frac{1}{3}$ | |
| Sol. | (D) $\pm \frac{1}{3}$ | 1 |
| 10. | A linear programming problem deals with the optimization of a/an :<br>(A) logarithmic function (B) linear function<br>(C) quadratic function (D) exponential function | |
| Sol. | (B) linear function | 1 |
| 11. | If $P(A \mid B) = P(A' \mid B)$, then which of the following statements is true ?<br>(A) $P(A) = P(A')$ (B) $P(A) = 2\,P(B)$<br>(C) $P(A \cap B) = \frac{1}{2}\,P(B)$ (D) $P(A \cap B) = 2\,P(B)$ | |
| Sol. | (C) $P(A \cap B) = \frac{1}{2} P(B)$ | 1 |
| 12. | $\begin{vmatrix} x+1 & x-1 \\ x^2+x+1 & x^2-x+1 \end{vmatrix}$ is equal to :<br>(A) $2x^3$ (B) 2<br>(C) 0 (D) $2x^3 - 2$ | |
| Sol. | (B) 2 | 1 |
| 13. | The derivative of $\sin(x^2)$ w.r.t. x, at $x = \sqrt{\pi}$ is :<br>(A) 1 (B) $-1$<br>(C) $-2\sqrt{\pi}$ (D) $2\sqrt{\pi}$ | |
| Sol. | (C) $-2\sqrt{\pi}$ | 1 |
| 14. | The order and degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \frac{d^2y}{dx^2}$ respectively are :<br>(A) 1, 2 (B) 2, 3<br>(C) 2, 1 (D) 2, 6 | |
| Sol. | (C) 2, 1 | 1 |

| | | |
|---|---|---|
| 15. | The vector with terminal point A (2, – 3, 5) and initial point B (3, – 4, 7) is :<br>(A) $\hat{i} - \hat{j} + 2\hat{k}$ (B) $\hat{i} + \hat{j} + 2\hat{k}$<br>(C) $-\hat{i} - \hat{j} - 2\hat{k}$ (D) $-\hat{i} + \hat{j} - 2\hat{k}$ | |
| Sol. | (D) $-\hat{\imath} + \hat{\jmath} - 2\hat{k}$ | 1 |
| 16. | The distance of point P(a, b, c) from y-axis is :<br>(A) b (B) $b^2$<br>(C) $\sqrt{a^2 + c^2}$ (D) $a^2 + c^2$ | |
| Sol. | (C) $\sqrt{a^2 + c^2}$ | 1 |
| 17. | The number of corner points of the feasible region determined by constraints $x \geq 0, y \geq 0, x + y \geq 4$ is :<br>(A) 0 (B) 1<br>(C) 2 (D) 3 | |
| Sol. | (C) 2 | 1 |
| 18. | If A and B are two non-zero square matrices of same order such that $(A + B)^2 = A^2 + B^2$, then :<br>(A) AB = O (B) AB = – BA<br>(C) BA = O (D) AB = BA | |
| Sol. | (B) AB = – BA | 1 |
| | Questions No. **19** & **20**, are Assertion (A) and Reason (R) based questions carrying **1** mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R).<br>Select the correct answer from the codes (A), (B), (C) and (D) as given below :<br>(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of Assertion (A).<br>(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).<br>(C) Assertion (A) is true, but Reason (R) is false.<br>(D) Assertion (A) is false, but Reason (R) is true. | |
| 19. | *Assertion (A) :* For matrix $A = \begin{bmatrix} 1 & \cos\theta & 1 \\ -\cos\theta & 1 & \cos\theta \\ -1 & -\cos\theta & 1 \end{bmatrix}$, where $\theta \in [0, 2\pi]$, $\lvert A \rvert \in [2, 4]$.<br>*Reason (R) :* $\cos\theta \in [-1, 1], \forall\, \theta \in [0, 2\pi]$. | |
| Sol. | (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of Assertion (A). | 1 |

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| 20. | *Assertion (A) :* A line in space cannot be drawn perpendicular to x, y and z axes simultaneously.<br><br>*Reason (R) :* For any line making angles, $\alpha$, $\beta$, $\gamma$ with the positive directions of x, y and z axes respectively, $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$. | |
| Sol. | (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of Assertion (A). | 1 |
| | **SECTION B**<br>In this section there are 5 very short answer type questions of 2 marks each. | |
| 21(a). | Check whether the function $f(x) = x^2 \|x\|$ is differentiable at x = 0 or not. | |
| Sol. | $f(x) = \begin{cases} x^3, x \geq 0 \\ -x^3, x \leq 0 \end{cases}$<br>$RHD = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0} h^2 = 0$<br>$LHD = \lim_{h\to 0} \frac{f(0-h)-f(0)}{-h} = \lim_{h\to 0} (-h^2) = 0$<br>$\because RHD = LHD = 0$, So $f(x)$ is differentiable at $x = 0$ | $\frac{1}{2}$<br>$\frac{1}{2}$<br>$\frac{1}{2}$<br>$\frac{1}{2}$ |
| | OR | |
| 21(b). | If $y = \sqrt{\tan\sqrt{x}}$, prove that $\sqrt{x}\frac{dy}{dx} = \frac{1+y^4}{4y}$. | |
| Sol. | $y = \sqrt{\tan\sqrt{x}}$<br>$\frac{dy}{dx} = \frac{\sec^2\sqrt{x}}{2\sqrt{\tan\sqrt{x}}} \times \frac{1}{2\sqrt{x}}$<br>$\sqrt{x}\frac{dy}{dx} = \frac{\sec^2\sqrt{x}}{4\sqrt{\tan\sqrt{x}}}$<br>$= \frac{1+(\tan\sqrt{x})^2}{4\sqrt{\tan\sqrt{x}}} = \frac{1+y^4}{4y}$ | 1<br><br>1 |
| 22. | Show that the function $f(x) = 4x^3 - 18x^2 + 27x - 7$ has neither maxima nor minima. | |
| Sol. | $f'(x) = 12x^2 - 36x + 27$<br>$= 3(2x-3)^2 \geq 0$ for all $x \in R$<br>$\therefore$ f is increasing on R .<br>Hence f(x) does not have maxima or minima. | $\frac{1}{2}$<br>$\frac{1}{2}$<br>$\frac{1}{2}$<br>$\frac{1}{2}$ |

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| 23(a). | Find : $\int x\sqrt{1+2x}\,dx$ | |
| Sol. | $1+2x=t^2$<br>$2\,dx=2t\,dt$<br>$\frac{1}{2}\int(t^4-t^2)dt=\frac{1}{2}\left[\frac{t^5}{5}-\frac{t^3}{3}\right]+C$<br>$=\frac{(1+2x)^{\frac{5}{2}}}{10}-\frac{(1+2x)^{\frac{3}{2}}}{6}+C$ | $\frac{1}{2}$<br>1<br>$\frac{1}{2}$ |
| | OR | |
| 23(b). | Evaluate : $\int_0^{\frac{\pi^2}{4}}\frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$ | |
| Sol. | $\int_0^{\frac{\pi^2}{4}}\frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$ Put $\sqrt{x}=t \Rightarrow dx=2t\,dt$<br>$2\int_0^{\frac{\pi}{2}}\sin t\,dt=2\,[-\cos t]_0^{\frac{\pi}{2}}$<br>$=2$ | $\frac{1}{2}$<br>1<br>$\frac{1}{2}$ |
| 24. | If $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that $(\vec{a}+\vec{b})\perp\vec{a}$ and $(2\vec{a}+\vec{b})\perp\vec{b}$, then prove that $\lvert\vec{b}\rvert=\sqrt{2}\,\lvert\vec{a}\rvert$. | |
| Sol. | $(\vec{a}+\vec{b}).\vec{a}=0 \Rightarrow \lvert\vec{a}\rvert^2+\vec{b}.\vec{a}=0$ -----------------(1)<br>$(2\vec{a}+\vec{b}).\vec{b}=0 \Rightarrow 2\vec{a}.\vec{b}+\lvert\vec{b}\rvert^2=0$ -------------(2)<br>$2(-\lvert\vec{a}\rvert^2)+\lvert\vec{b}\rvert^2=0$ {Using (1) and (2)}<br>$\lvert\vec{b}\rvert^2=2\lvert\vec{a}\rvert^2 \Rightarrow \lvert\vec{b}\rvert=\sqrt{2}\lvert\vec{a}\rvert$ | $\frac{1}{2}$<br>$\frac{1}{2}$<br>$\frac{1}{2}$<br>$\frac{1}{2}$ |

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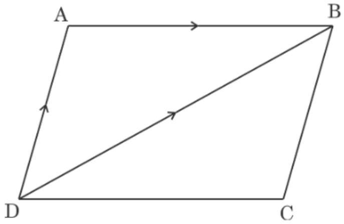
| 25. | In the given figure, ABCD is a parallelogram. If $\overrightarrow{AB} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\overrightarrow{DB} = 3\hat{i} - 6\hat{j} + 2\hat{k}$, then find $\overrightarrow{AD}$ and hence find the area of parallelogram ABCD.<br><br> | |
| Sol. | $\overrightarrow{AD} + \overrightarrow{DB} = \overrightarrow{AB}$<br>$\overrightarrow{AD} = (2\hat{\imath} - 4\hat{\jmath} + 5\hat{k}) - (3\hat{\imath} - 6\hat{\jmath} + 2\hat{k})$<br>$= -\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$<br>$\overrightarrow{AD} \times \overrightarrow{AB} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 2 & 3 \\ 2 & -4 & 5 \end{vmatrix} = 22\hat{\imath} + 11\hat{\jmath}$<br>Area $= \lvert\overrightarrow{AD} \times \overrightarrow{AB}\rvert = \lvert 22\hat{\imath} + 11\hat{\jmath}\rvert$<br>$= \sqrt{605}$ *or* $11\sqrt{5}$ | $\frac{1}{2}$<br>1<br>$\frac{1}{2}$ |
| | **SECTION C**<br>In this section there are 6 short answer type questions of 3 marks each. | |
| 26(a). | A relation R on set A = {1, 2, 3, 4, 5} is defined as $R = \{(x, y) : \lvert x^2 - y^2\rvert < 8\}$. Check whether the relation R is reflexive, symmetric and transitive. | |
| Sol. | (a) Reflexive:<br>$\because \lvert x^2 - x^2\rvert < 8 \; \forall \; x \in A \Rightarrow (x, x) \in R \;\; \therefore R$ is reflexive .<br>(b) Symmetric:<br>Let (x,y) ∈ $R$ for some x,y ∈ A<br>$\therefore \; \lvert x^2 - y^2\rvert < 8 \Rightarrow \lvert y^2 - x^2\rvert < 8 \Rightarrow (y, x) \in R$<br>Hence R is symmetric.<br>(c) Transitive:<br>(1,2) , (2,3) ∈ $R$ as $\lvert 1^2 - 2^2\rvert < 8$ , $\lvert 2^2 - 3^2\rvert < 8$ respectively<br>But $\lvert 1^2 - 3^2\rvert \nless 8 \Rightarrow (1, 3) \notin R$<br>Hence $R$ is not transitive. | $\frac{1}{2}$<br>1<br>$1\frac{1}{2}$ |
| | OR | |
| 26(b). | A function f is defined from R → R as f(x) = ax + b, such that f(1) = 1 and f(2) = 3. Find function f(x). Hence, check whether function f(x) is one-one and onto or not. | |

| | | |
|---|---|---|
| Sol. | $f(x) = ax + b$<br>Solving $a + b = 1$ and $2a + b = 3$ to get $a=2$ , $b = -1$<br>$f(x) = 2x - 1$<br>Let $f(x_1) = f(x_2)$ for some $x_1, x_2 \in \mathbf{R}$<br>$2x_1 - 1 = 2x_2 - 1 \Rightarrow x_1 = x_2$<br>Hence f is one – one.<br>Let $y = 2x - 1$, $y \in \mathbf{R}$ (Codomain)<br>$\Rightarrow x = \frac{y+1}{2} \in \mathbf{R}$ (domain)<br>Also, $f(x) = f\left(\frac{y+1}{2}\right) = y$<br>$\therefore$ f is onto. | <br>1<br><br><br><br>1<br><br><br><br>1 |
| 27(a). | If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$. | |
| Sol. | $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$<br>Put $x = \sin\theta$ , $y = \sin\phi$<br>$\Rightarrow \cos\theta + \cos\phi = a(\sin\theta - \sin\phi)$<br>$\Rightarrow 2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right) = 2a\sin\left(\frac{\theta-\phi}{2}\right)\cos\left(\frac{\theta+\phi}{2}\right)$<br>$\Rightarrow \cot\left(\frac{\theta-\phi}{2}\right) = a$<br>$\Rightarrow \theta - \phi = 2\cot^{-1} a$<br>$\Rightarrow \sin^{-1} x - \sin^{-1} y = 2\cot^{-1} a$<br>$\Rightarrow \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$<br>$\Rightarrow \frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$ | $\frac{1}{2}$<br>$\frac{1}{2}$<br><br>$\frac{1}{2}$<br>1<br>$\frac{1}{2}$ |
| | OR | |
| 27(b). | If $y = (\tan x)^x$, then find $\frac{dy}{dx}$. | |
| Sol. | $y = (\tan x)^x$<br>$\log y = x \log(\tan x)$<br>$\frac{1}{y}\frac{dy}{dx} = x\left(\frac{\sec^2 x}{\tan x}\right) + \log(\tan x)$<br>$\frac{dy}{dx} = (\tan x)^x\left[\left(\frac{x\sec^2 x}{\tan x}\right) + \log(\tan x)\right]$ | $\frac{1}{2}$<br>2<br>$\frac{1}{2}$ |
| 28(a). | Find :<br>$\int \frac{x^2}{(x^2+4)(x^2+9)}\,dx$ | |

| | | |
|---|---|---|
| Sol. | Let $I = \int \frac{x^2}{(x^2+4)(x^2+9)} dx$<br>Put $x^2 = t$<br>$\frac{t}{(t+4)(t+9)} = \frac{A}{t+4} + \frac{B}{t+9} \Rightarrow A = \frac{-4}{5}, \ B = \frac{9}{5}$<br>$I = \frac{-4}{5}\int \frac{1}{2^2+x^2}\, dx + \frac{9}{5}\int \frac{1}{3^2+x^2}\, dx$<br>$= \frac{-2}{5}\tan^{-1}\left(\frac{x}{2}\right) + \frac{3}{5}\tan^{-1}\left(\frac{x}{3}\right) + C$ | $\frac{1}{2}$<br>$1\frac{1}{2}$<br>1 |
| | OR | |
| 28(b). | Evaluate :<br>$$\int_1^3 (\lvert x-1\rvert + \lvert x-2\rvert + \lvert x-3\rvert)\, dx$$ | |
| Sol. | $\int_1^3 (\lvert x-1\rvert + \lvert x-2\rvert + \lvert x-3\rvert) dx$<br>$= \int_1^3 (x-1)dx + \int_1^2 -(x-2)dx + \int_2^3 (x-2)dx - \int_1^3 (x-3)dx$<br>$= \int_1^3 2\, dx + \int_1^2 (2-x)\, dx + \int_2^3 (x-2)dx$<br>$= [2x]_1^3 + \left[\frac{(2-x)^2}{-2}\right]_1^2 + \left[\frac{(x-2)^2}{2}\right]_2^3$<br>$= 4 + \frac{1}{2} + \frac{1}{2} = 5$ | $1\frac{1}{2}$<br>$1\frac{1}{2}$ |
| 29. | Find the particular solution of the differential equation given by $x^2 \frac{dy}{dx} - xy = x^2 \cos^2\left(\frac{y}{2x}\right)$, given that when $x = 1, y = \frac{\pi}{2}$. | |
| Sol. | $\frac{dy}{dx} = \frac{y}{x} + \cos^2(\frac{y}{2x})$<br>Put $y = vx$ so that $\frac{dy}{dx} = v + x\frac{dv}{dx}$<br>$\Rightarrow v + x\frac{dv}{dx} = v + \cos^2(\frac{v}{2})$<br>$\Rightarrow \int \sec^2\left(\frac{v}{2}\right) dv = \int \frac{1}{x}\, dx$<br>$\Rightarrow 2\tan\left(\frac{v}{2}\right) = \log\lvert x\rvert + C$<br>$\Rightarrow 2\tan\left(\frac{y}{2x}\right) = \log\lvert x\rvert + C$<br>$2\tan\frac{\pi}{4} = \log 1 + C \Rightarrow C = 2 \Rightarrow 2\tan\left(\frac{y}{2x}\right) = \log\lvert x\rvert + 2$ | $\frac{1}{2}$<br>$\frac{1}{2}$<br>$\frac{1}{2}$<br>1<br>$\frac{1}{2}$ |

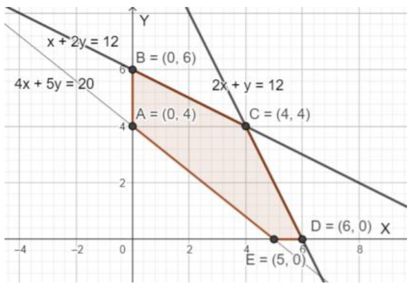
**30.** Solve the following linear programming problem graphically :

Maximise $z = 500x + 300y$,

subject to constraints

$$x + 2y \le 12$$
$$2x + y \le 12$$
$$4x + 5y \ge 20$$
$$x \ge 0, y \ge 0$$

**Sol.** Max $z = 500x + 300y$




Correct Graph - $1\frac{1}{2}$

| Corner Point | Z |
|---|---|
| A (0,4) | 1200 |
| B (0,6) | 1800 |
| C (4,4) | 3200 |
| D (6,0) | 3000 |
| E (5,0) | 2500 |

Correct Table - 1

Max $z = 3200$ at $x = 4, y = 4$ — $\frac{1}{2}$

**31.** E and F are two independent events such that $P(\bar{E}) = 0{\cdot}6$ and $P(E \cup F) = 0{\cdot}6$. Find $P(F)$ and $P(\bar{E} \cup \bar{F})$.

**Sol.** $P(\bar{E}) = 0.6 \Rightarrow P(E) = 0.4$ — $\frac{1}{2}$

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$ — $\frac{1}{2}$

$\Rightarrow 0.6 = 0.4 + P(F) - 0.4\,P(F) \Rightarrow P(F) = \frac{1}{3}$ — 1

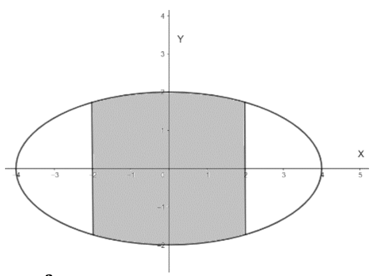
$P(\bar{E} \cup \bar{F}) = 1 - P(E \cap F)$ — $\frac{1}{2}$

$= 1 - 0.4 \times \frac{1}{3} = \frac{13}{15}$ — $\frac{1}{2}$

## SECTION D

In this section there are 4 long answer type questions of 5 marks each.

| | | |
|---|---|---|
| 32(a). | If $A=\begin{bmatrix}1 & -2 & 0\\ 2 & -1 & -1\\ 0 & -2 & 1\end{bmatrix}$, find $A^{-1}$ and use it to solve the following system of equations : <br> $x-2y=10,\ 2x-y-z=8,\ -2y+z=7$ | |
| Sol. | $\lvert A\rvert = 1 \neq 0$ hence $A^{-1}$ exists. <br> $Adj\ A = \begin{bmatrix}-3 & 2 & 2\\ -2 & 1 & 1\\ -4 & 2 & 3\end{bmatrix}$ <br> $A^{-1} = \begin{bmatrix}-3 & 2 & 2\\ -2 & 1 & 1\\ -4 & 2 & 3\end{bmatrix}$ <br> $AX = B \Rightarrow \begin{bmatrix}1 & -2 & 0\\ 2 & -1 & -1\\ 0 & -2 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}10\\ 8\\ 7\end{bmatrix}$ <br> $X = A^{-1}B \Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}-3 & 2 & 2\\ -2 & 1 & 1\\ -4 & 2 & 3\end{bmatrix}\begin{bmatrix}10\\ 8\\ 7\end{bmatrix} = \begin{bmatrix}0\\ -5\\ -3\end{bmatrix}$ <br> $\Rightarrow x = 0, y = -5, z = -3$ | 1 <br> 2 <br> $\frac{1}{2}$ <br> $1\frac{1}{2}$ |
| | OR | |
| 32(b). | If $A=\begin{bmatrix}-1 & a & 2\\ 1 & 2 & x\\ 3 & 1 & 1\end{bmatrix}$ and $A^{-1}=\begin{bmatrix}1 & -1 & 1\\ -8 & 7 & -5\\ b & y & 3\end{bmatrix}$, find the value of $(a+x)-(b+y)$. | |
| Sol. | $AA^{-1} = I$ <br> $\begin{bmatrix}-1 & a & 2\\ 1 & 2 & x\\ 3 & 1 & 1\end{bmatrix}\begin{bmatrix}1 & -1 & 1\\ -8 & 7 & -5\\ b & y & 3\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ <br> $\begin{bmatrix}-1-8a+2b & 1+7a+2y & 5-5a\\ -15+bx & 13+xy & 3x-9\\ -5+b & 4+y & 1\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ <br> $-5+b=0 \Rightarrow b=5$, $5-5a=0 \Rightarrow a=1$ <br> $4+y=0 \Rightarrow y=-4$, $3x-9=0 \Rightarrow x=3$ <br> $\therefore (a+x)-(b+y) = (1+3)-(5-4) = 3$ | 1 <br> $1\frac{1}{2}$ <br> 1 <br> 1 <br> $\frac{1}{2}$ |
| 33(a). | Evaluate : <br> $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16\sin 2x}\,dx$ | |
| Sol. | Let $I = \int_0^{\frac{\pi}{4}} \frac{\sin x+\cos x}{9+16\sin 2x}\,dx$ | |

| | | |
|---|---|---|
| | Put sin x – cos x = t, so that (cos x + sin x) dx = dt<br>$sin^2x + cos^2x - sin2x = t^2 \Rightarrow sin2x = 1 - t^2$<br>$I = \int_{-1}^{0} \frac{dt}{25-16t^2}$<br>$= \frac{1}{16}\int_{-1}^{0} \frac{dt}{(\frac{5}{4})^2 - t^2}$<br>$= \frac{1}{40}\left[log\left\|\frac{5+4t}{5-4t}\right\|\right]_{-1}^{0}$<br>$= \frac{1}{40}\left[\log 1 - \log\left(\frac{1}{9}\right)\right] = \frac{1}{40}\log 9 \ or\ \frac{1}{20}\log 3$ | 1<br>$\frac{1}{2}$<br>1<br>$1\frac{1}{2}$<br>1 |
| | OR | |
| 33(b). | Evaluate :<br>$\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x)\, dx$ | |
| Sol. | $Let\ I = \int_0^{\frac{\pi}{2}} \sin 2x\ \tan^{-1}(sin\ x)\ dx$<br>Put sin x = t so that cos x dx = dt<br>$I = 2\int_0^1 t\ \tan^{-1} t\ dt$<br>$= 2\left[\tan^{-1} t\left(\frac{t^2}{2}\right) - \frac{1}{2}\int \frac{t^2}{1+t^2} dt\right]_0^1$<br>$= 2\left[\left(\frac{t^2}{2}\right)\tan^{-1} t - \frac{1}{2}t + \frac{1}{2}\tan^{-1} t\right]_0^1$<br>$= 2\left(\frac{\pi}{4} - \frac{1}{2}\right) = \frac{\pi}{2} - 1$ | 1<br>1<br>$1\frac{1}{2}$<br>1<br>$\frac{1}{2}$ |
| 34. | Using integration, find the area of the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$, included between the lines x = – 2 and x = 2. | |
| Sol. | <br>Area $= 4\int_0^2 y\ dx$<br>$= 4\left[\frac{1}{2}\int_0^2 \sqrt{4^2 - x^2}\ dx\right]$<br>$= 2\left[\frac{x}{2}\sqrt{4^2 - x^2} + 8\ \sin^{-1}\left(\frac{x}{4}\right)\right]_0^2$<br>$= 2\left[\sqrt{12} + \frac{8\pi}{6}\right] = 4\sqrt{3} + \frac{8\pi}{3}$ | Correct graph-1<br>1<br>2<br>1 |

| | | |
|---|---|---|
| | | |
| 35. | The image of point P(x, y, z) with respect to line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ is $P'(1, 0, 7)$. Find the coordinates of point P. | |

**Sol.**

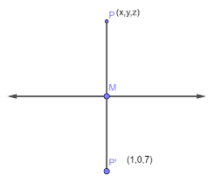
Let foot of the perpendicular on the given line from point P be M $(\lambda, 2\lambda+1, 3\lambda+2)$ — **1**

D. ratios of $PP'$ are $\lambda - 1,\ 2\lambda+1,\ 3\lambda-5$ — **1**

$1(\lambda - 1) + 2(2\lambda+1) + 3(3\lambda - 5) = 0$

$\Rightarrow \lambda = 1$ — **1**

Coordinates of M(1,3,5) — **1**



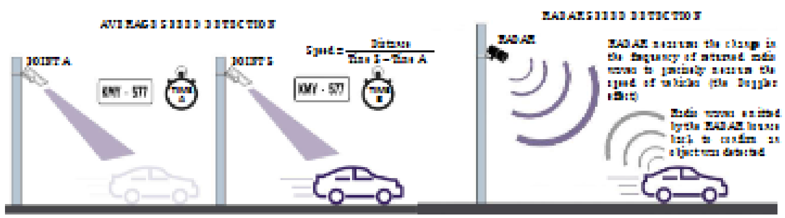

$\frac{x+1}{2} = 1,\ \frac{y+0}{2} = 3,\ \frac{z+7}{2} = 5$

$\Rightarrow x = 1,\ y = 6,\ z = 3 \Rightarrow P(1, 6, 3)$ — **1**

## SECTION E

In this section there are 3 case-study based questions of 4 marks each.

**36.** The traffic police has installed Over Speed Violation Detection (OSVD) system at various locations in a city. These cameras can capture a speeding vehicle from a distance of 300 m and even function in the dark.




A camera is installed on a pole at the height of 5 m. It detects a car travelling away from the pole at the speed of 20 m/s. At any point, x m away from the base of the pole, the angle of elevation of the speed camera from the car C is θ.

On the basis of the above information, answer the following questions :

(i) Express θ in terms of height of the camera installed on the pole and x. — *1*

(ii) Find $\frac{d\theta}{dx}$. — *1*

(iii) (a) Find the rate of change of angle of elevation with respect to time at an instant when the car is 50 m away from the pole. — *2*

**OR**

(iii) (b) If the rate of change of angle of elevation with respect to time of another car at a distance of 50 m from the base of the pole is $\frac{3}{101}$ rad/s, then find the speed of the car. — *2*

**Sol.**

(i) $\tan\theta = \frac{5}{x} \Rightarrow \theta = \tan^{-1}\left(\frac{5}{x}\right)$ — **1**

(ii) $\frac{d\theta}{dx} = \frac{-5}{5^2 + x^2}$ — **1**

| | | |
|---|---|---|
| | (iii) (a) $\frac{d\theta}{dt} = \frac{d\theta}{dx} \times \frac{dx}{dt} = \frac{-5}{5^2+x^2} \times 20\Big]_{x=50}$<br>$= \frac{-100}{2525}$ or $\frac{-4}{101}$ $rad/s$<br>OR<br>(b) $\frac{d\theta}{dt} = \frac{d\theta}{dx} \times \frac{dx}{dt} \Rightarrow \frac{3}{101} = \frac{-5}{5^2+x^2}\Big]_{x=50} \times \frac{dx}{dt}$<br>$\Rightarrow \frac{3}{101} = \frac{-5}{2525} \times \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = -15\ m/s$<br>Hence the speed is 15 m/s | $1\frac{1}{2}$<br>$\frac{1}{2}$<br><br>$1\frac{1}{2}$<br>$\frac{1}{2}$ |
| 37. | According to recent research, air turbulence has increased in various regions around the world due to climate change. Turbulence makes flights bumpy and often delays the flights.<br><br>Assume that, an airplane observes severe turbulence, moderate turbulence or light turbulence with equal probabilities. Further, the chance of an airplane reaching late to the destination are 55%, 37% and 17% due to severe, moderate and light turbulence respectively.<br><br><br><br><br>On the basis of the above information, answer the following questions :<br>(i) Find the probability that an airplane reached its destination late. *2*<br>(ii) If the airplane reached its destination late, find the probability that it was due to moderate turbulence. *2* | |
| Sol. | (i) Let A denote the event of airplane reaching its destination late<br>$E_1$ = severe turbulence<br>$E_2$ = moderate turbulence<br>$E_3$ = light turbulence<br>$P(A) = P(E_1)\,P(A\mid E_1) + P(E_2)P(A\mid E_2) + P(E_3)P(A\mid E_3)$<br>$= \frac{1}{3} \times \frac{55}{100} + \frac{1}{3} \times \frac{37}{100} + \frac{1}{3} \times \frac{17}{100}$<br>$= \frac{1}{3}\left(\frac{109}{100}\right) = \frac{109}{300}$ | $\left.\right\}\frac{1}{2}$<br><br><br>1<br>$\frac{1}{2}$ |

(ii) $P(E_2|A) = \frac{P(E_2)P(A|E_2)}{P(A)}$

$= \frac{\frac{1}{3} \times \frac{37}{100}}{\frac{109}{300}}$

$1\frac{1}{2}$

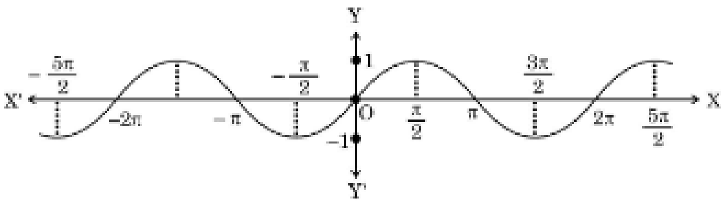
$= \frac{37}{109}$

$\frac{1}{2}$

**38.**

If a function f : X → Y defined as f(x) = y is one-one and onto, then we can define a unique function g : Y → X such that g(y) = x, where x ∈ X and y = f(x), y ∈ Y. Function g is called the inverse of function f.

The domain of sine function is R and function sine : R → R is neither one-one nor onto. The following graph shows the sine function.




y = sin x

Let sine function be defined from set A to [– 1, 1] such that inverse of sine function exists, i.e., $\sin^{-1} x$ is defined from [– 1, 1] to A.

On the basis of the above information, answer the following questions :

(i) If A is the interval other than principal value branch, give an example of one such interval. *1*

(ii) If $\sin^{-1}(x)$ is defined from [– 1, 1] to its principal value branch, find the value of $\sin^{-1}\left(-\frac{1}{2}\right) - \sin^{-1}(1)$. *1*

(iii) (a) Draw the graph of $\sin^{-1} x$ from [– 1, 1] to its principal value branch. *2*

**OR**

(iii) (b) Find the domain and range of $f(x) = 2\sin^{-1}(1 - x)$. *2*

**Sol.**

(i) $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ or any other interval corresponding to the domain [-1,1]

1

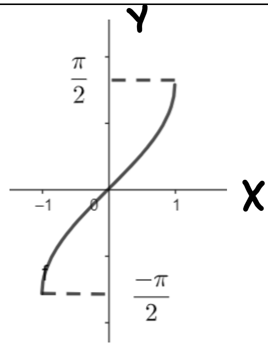
(ii) $\sin^{-1}\left(\frac{-1}{2}\right) - \sin^{-1}(1)$

$= \frac{-\pi}{6} - \frac{\pi}{2}$

$= \frac{-4\pi}{6}$ or $\frac{-2\pi}{3}$

1

(iii) (a)




Correct graph - 2

OR

(b) $f(x) = 2\, sin^{-1}(1 - x)$

$-1 \leq 1 - x \leq 1$ — $\frac{1}{2}$

$\Rightarrow -2 \leq -x \leq 0$

$\Rightarrow 0 \leq x \leq 2$

Domain $= [0, 2]$ — $\frac{1}{2}$

$\frac{-\pi}{2} \leq \sin^{-1}(1 - x) \leq \frac{\pi}{2}$ — $\frac{1}{2}$

$-\pi \leq 2 \sin^{-1}(1 - x) \leq \pi$

So range $= [-\pi, \pi]$ — $\frac{1}{2}$