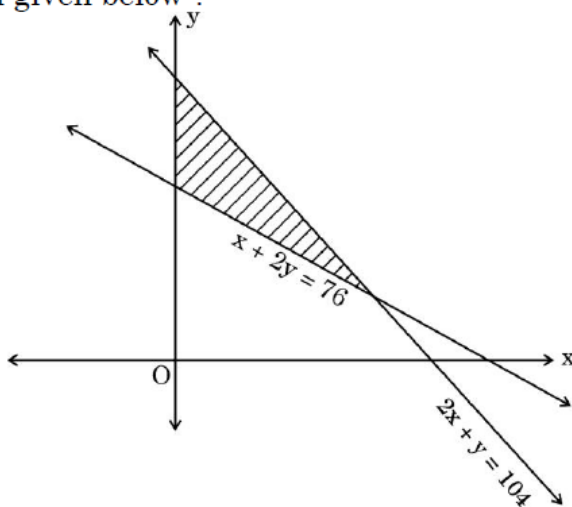


Q. NO.	EXPECTED ANSWER / VALUE POINT	MARKS
<b>SECTION A</b>		
Questions no. 1 to 18 are multiple choice questions (MCQs) of 1 mark each.		
Q1	<p>If <math>A = [a_{ij}]</math> is an identity matrix, then which of the following is true ?</p> <p>(A) <math>a_{ij} = \begin{cases} 0, &amp; \text{if } i=j \\ 1, &amp; \text{if } i \neq j \end{cases}</math>                      (B) <math>a_{ij} = 1, \forall i, j</math></p> <p>(C) <math>a_{ij} = 0, \forall i, j</math>                      (D) <math>a_{ij} = \begin{cases} 0, &amp; \text{if } i \neq j \\ 1, &amp; \text{if } i = j \end{cases}</math></p>	
Ans	(D) $a_{ij} = \begin{cases} 0, & \text{if } i \neq j \\ 1, & \text{if } i = j \end{cases}$	1
Q2	<p>Let <math>R_+</math> denote the set of all non-negative real numbers. Then the function <math>f: R_+ \rightarrow R_+</math> defined as <math>f(x) = x^2 + 1</math> is :</p> <p>(A) one-one but not onto                      (B) onto but not one-one</p> <p>(C) both one-one and onto                      (D) neither one-one nor onto</p>	
Ans	(A) one-one but not onto	1
Q3	<p>Let <math>A = \begin{bmatrix} a &amp; b \\ c &amp; d \end{bmatrix}</math> be a square matrix such that <math>\text{adj } A = A</math>. Then, <math>(a + b + c + d)</math> is equal to :</p> <p>(A) <math>2a</math>                      (B) <math>2b</math></p> <p>(C) <math>2c</math>                      (D) <math>0</math></p>	
Ans	(A) $2a$	1
Q4	<p>A function <math>f(x) =  1 - x +  x  </math> is :</p> <p>(A) discontinuous at <math>x = 1</math> only                      (B) discontinuous at <math>x = 0</math> only</p> <p>(C) discontinuous at <math>x = 0, 1</math>                      (D) continuous everywhere</p>	
Ans	(D) continuous everywhere	1
Q5	<p>If the sides of a square are decreasing at the rate of <math>1.5</math> cm/s, the rate of decrease of its perimeter is :</p> <p>(A) <math>1.5</math> cm/s                      (B) <math>6</math> cm/s</p> <p>(C) <math>3</math> cm/s                      (D) <math>2.25</math> cm/s</p>	
Ans	(B) $6$ cm/s	1



Ans	(D) 0	1
Q12	<p>If A and B are two skew symmetric matrices, then (AB + BA) is :</p> <p>(A) a skew symmetric matrix      (B) a symmetric matrix  (C) a null matrix                      (D) an identity matrix</p>	
Ans	(B) a symmetric matrix	1
Q13	<p>If <math>\begin{vmatrix} 1 &amp; 3 &amp; 1 \\ k &amp; 0 &amp; 1 \\ 0 &amp; 0 &amp; 1 \end{vmatrix} = \pm 6</math>, then the value of k is :</p> <p>(A) 2                      (B) -2                      (C) <math>\pm 2</math>                      (D) <math>\mp 2</math></p>	
Ans	(D) $\mp 2$	1
Q14	<p>The derivative of <math>2^x</math> w.r.t. <math>3^x</math> is :</p> <p>(A) <math>\left(\frac{3}{2}\right)^x \frac{\log 2}{\log 3}</math>                      (B) <math>\left(\frac{2}{3}\right)^x \frac{\log 3}{\log 2}</math>  (C) <math>\left(\frac{2}{3}\right)^x \frac{\log 2}{\log 3}</math>                      (D) <math>\left(\frac{3}{2}\right)^x \frac{\log 3}{\log 2}</math></p>	
Ans	(C) $\left(\frac{2}{3}\right)^x \frac{\log 2}{\log 3}$	1
Q15	<p>If <math> \vec{a}  = 2</math> and <math>-3 \leq k \leq 2</math>, then <math> k\vec{a}  \in</math> :</p> <p>(A) [-6, 4]                      (B) [0, 4]  (C) [4, 6]                      (D) [0, 6]</p>	
Ans	(D) [0, 6]	1
Q16	<p>If a line makes an angle of <math>\frac{\pi}{4}</math> with the positive directions of both x-axis and z-axis, then the angle which it makes with the positive direction of y-axis is :</p> <p>(A) 0                      (B) <math>\frac{\pi}{4}</math>                      (C) <math>\frac{\pi}{2}</math>                      (D) <math>\pi</math></p>	
Ans	(C) $\frac{\pi}{2}$	1

Q17	<p>Of the following, which group of constraints represents the feasible region given below ?</p>  <p>(A) <math>x + 2y \leq 76, 2x + y \geq 104, x, y \geq 0</math>  (B) <math>x + 2y \leq 76, 2x + y \leq 104, x, y \geq 0</math>  (C) <math>x + 2y \geq 76, 2x + y \leq 104, x, y \geq 0</math>  (D) <math>x + 2y \geq 76, 2x + y \geq 104, x, y \geq 0</math></p>	
Ans	(C) $x + 2y \geq 76, 2x + y \leq 104, x, y \geq 0$	1
Q18	<p>If <math>A = \begin{bmatrix} 2 &amp; 0 &amp; 0 \\ 0 &amp; 3 &amp; 0 \\ 0 &amp; 0 &amp; 5 \end{bmatrix}</math>, then <math>A^{-1}</math> is :</p> <p>(A) <math>\begin{bmatrix} \frac{1}{2} &amp; 0 &amp; 0 \\ 0 &amp; \frac{1}{3} &amp; 0 \\ 0 &amp; 0 &amp; \frac{1}{5} \end{bmatrix}</math>      (B) <math>30 \begin{bmatrix} \frac{1}{2} &amp; 0 &amp; 0 \\ 0 &amp; \frac{1}{3} &amp; 0 \\ 0 &amp; 0 &amp; \frac{1}{5} \end{bmatrix}</math>  (C) <math>\frac{1}{30} \begin{bmatrix} 2 &amp; 0 &amp; 0 \\ 0 &amp; 3 &amp; 0 \\ 0 &amp; 0 &amp; 5 \end{bmatrix}</math>      (D) <math>\frac{1}{30} \begin{bmatrix} \frac{1}{2} &amp; 0 &amp; 0 \\ 0 &amp; \frac{1}{3} &amp; 0 \\ 0 &amp; 0 &amp; \frac{1}{5} \end{bmatrix}</math></p>	
Ans	(A) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$	1

Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Assertion (A) is false, but Reason (R) is true.

Q19	<p>Assertion (A) : Every scalar matrix is a diagonal matrix.</p> <p>Reason (R) : In a diagonal matrix, all the diagonal elements are 0.</p>	
Ans	(C) Assertion (A) is true, but Reason (R) is false.	1
Q20	<p>Assertion (A) : Projection of <math>\vec{a}</math> on <math>\vec{b}</math> is same as projection of <math>\vec{b}</math> on <math>\vec{a}</math>.</p> <p>Reason (R) : Angle between <math>\vec{a}</math> and <math>\vec{b}</math> is same as angle between <math>\vec{b}</math> and <math>\vec{a}</math> numerically.</p>	
Ans	(D) Assertion (A) is false, but Reason (R) is true.	1
<b>SECTION B</b>		
Questions no. 21 to 25 are very short answer (VSA) type questions, carrying 2 marks each.		
Q21	<p>Evaluate :</p> $\sec^2\left(\tan^{-1}\frac{1}{2}\right) + \operatorname{cosec}^2\left(\cot^{-1}\frac{1}{3}\right)$	
Ans	$\sec^2\left(\tan^{-1}\frac{1}{2}\right) + \operatorname{cosec}^2\left(\cot^{-1}\frac{1}{3}\right)$ $= \left[1 + \tan^2\left(\tan^{-1}\frac{1}{2}\right)\right] + \left[1 + \cot^2\left(\cot^{-1}\frac{1}{3}\right)\right]$ $= \left[1 + \left(\frac{1}{2}\right)^2\right] + \left[1 + \left(\frac{1}{3}\right)^2\right]$ $= \frac{85}{36}$	<p>1</p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p>
Q22(a)	If $x = e^{x/y}$ , prove that $\frac{dy}{dx} = \frac{\log x - 1}{(\log x)^2}$	

Ans	$x = e^{\frac{x}{y}} \Rightarrow \log x = \frac{x}{y} \Rightarrow y = \frac{x}{\log x}$ $\Rightarrow \frac{dy}{dx} = \frac{(\log x)(1) - x\left(\frac{1}{x}\right)}{(\log x)^2} = \frac{\log x - 1}{(\log x)^2}$	1 1
OR		
Q22(b)	Check the differentiability of $f(x) = \begin{cases} x^2 + 1, & 0 \leq x < 1 \\ 3 - x, & 1 \leq x \leq 2 \end{cases}$ at $x = 1$ .	
Ans	<p>LHD at <math>x = 1</math></p> $= \lim_{h \rightarrow 0} \frac{f(1-h) - f(1)}{-h} = \lim_{h \rightarrow 0} \frac{[(1-h)^2 + 1] - 2}{-h} = 2$ <p>RHD at <math>x = 1</math></p> $= \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \rightarrow 0} \frac{[3 - (1+h)] - 2}{h} = -1$ <p>as LHD <math>\neq</math> RHD, so <math>f(x)</math> is not differentiable at <math>x = 1</math></p>	1  1/2 1/2
Q23(a)	Evaluate :	
	$\int_0^{\pi/2} \sin 2x \cos 3x \, dx$	
Ans	$I = \int_0^{\pi/2} \sin 2x \cos 3x \, dx$ $= \frac{1}{2} \int_0^{\pi/2} (\sin 5x - \sin x) \, dx$ $= \frac{1}{2} \left[ -\frac{1}{5} \cos 5x + \cos x \right]_0^{\pi/2}$ $= -\frac{2}{5}$	1  1/2 1/2
OR		
Q23(b)	Given $\frac{d}{dx} F(x) = \frac{1}{\sqrt{2x - x^2}}$ and $F(1) = 0$ , find $F(x)$ .	

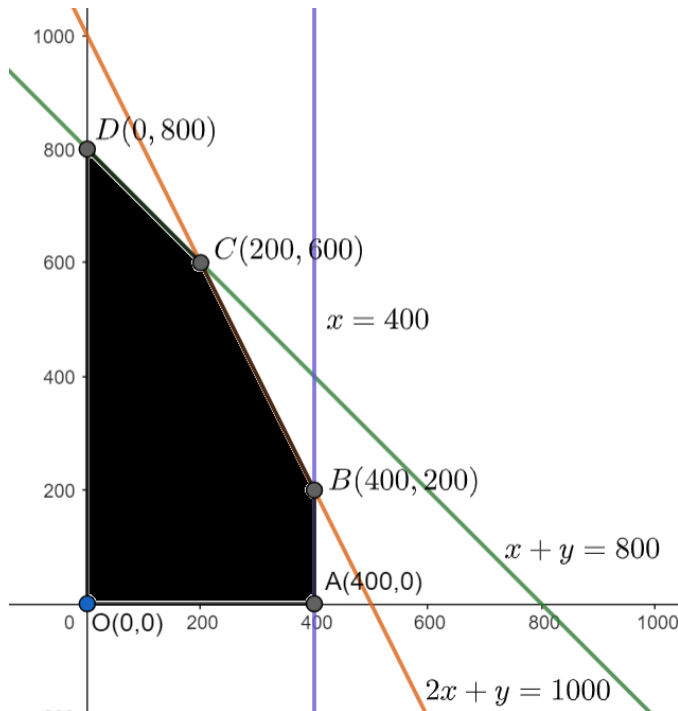
Ans	$F(x) = \int \frac{1}{\sqrt{2x-x^2}} dx$ $= \int \frac{1}{\sqrt{1-(x-1)^2}} dx$ $= \sin^{-1}(x-1) + c$ <p>when <math>x=1, y=0</math> gives <math>c=0</math></p> $\therefore F(x) = \sin^{-1}(x-1)$	<p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>
Q24	<p>Find the position vector of point C which divides the line segment joining points A and B having position vectors <math>\hat{i} + 2\hat{j} - \hat{k}</math> and <math>-\hat{i} + \hat{j} + \hat{k}</math> respectively in the ratio 4 : 1 externally. Further, find <math> \vec{AB}  :  \vec{BC} </math>.</p>	
Ans	<p>Position vector of C <math>= \vec{r} = \frac{4\vec{b} - \vec{a}}{3}</math></p> <p>i.e. <math>\vec{r} = \frac{1}{3}(-5\hat{i} + 2\hat{j} + 5\hat{k})</math></p> <p>Now, <math>\vec{AB} = -2\hat{i} - \hat{j} + 2\hat{k} \Rightarrow  \vec{AB}  = 3</math></p> <p><math>\vec{BC} = -\frac{1}{3}(2\hat{i} + \hat{j} - 2\hat{k}) \Rightarrow  \vec{BC}  = 1</math></p> <p><math> \vec{AB}  :  \vec{BC}  = 3:1</math></p>	<p>1</p> <p>1</p>
Q25	<p>Let <math>\vec{a}</math> and <math>\vec{b}</math> be two non-zero vectors.</p> <p>Prove that <math> \vec{a} \times \vec{b}  \leq  \vec{a}   \vec{b} </math>.</p> <p>State the condition under which equality holds, i.e., <math> \vec{a} \times \vec{b}  =  \vec{a}   \vec{b} </math>.</p>	
Ans	<p><math> \vec{a} \times \vec{b}  =  \vec{a}   \vec{b}   \sin \theta </math></p> <p>As, <math>0 \leq  \sin \theta  \leq 1</math></p> <p><math>\Rightarrow  \vec{a}   \vec{b}   \sin \theta  \leq  \vec{a}   \vec{b} </math></p> <p><math>\Rightarrow  \vec{a} \times \vec{b}  \leq  \vec{a}   \vec{b} </math></p> <p>For equality, <math>\sin \theta = 1 \Rightarrow \theta = \frac{\pi}{2} \Rightarrow \vec{a}</math> is perpendicular to <math>\vec{b}</math>.</p>	<p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>
<p><b>SECTION C</b></p> <p>Questions no. 26 to 31 are short answer (SA) type questions, carrying 3 marks each.</p>		

Q26(a)	If $x \cos(p + y) + \cos p \sin(p + y) = 0$ , prove that $\cos p \frac{dy}{dx} = -\cos^2(p + y)$ , where $p$ is a constant.	
Ans	$x \cos(p + y) + \cos p \sin(p + y) = 0$ $\Rightarrow x = \frac{-\cos p \sin(p + y)}{\cos(p + y)} \Rightarrow x = -\cos p \cdot \tan(p + y)$ $\Rightarrow \frac{dx}{dy} = -\cos p \cdot \sec^2(p + y)$ $\Rightarrow \frac{dy}{dx} = \frac{-1}{\cos p \cdot \sec^2(p + y)}$ $\Rightarrow \cos p \frac{dy}{dx} = -\cos^2(p + y)$	 1  1  $\frac{1}{2}$  $\frac{1}{2}$
OR		
Q26(b)	Find the value of $a$ and $b$ so that function $f$ defined as : $f(x) = \begin{cases} \frac{x-2}{ x-2 } + a, & \text{if } x < 2 \\ a + b, & \text{if } x = 2 \\ \frac{x-2}{ x-2 } + b, & \text{if } x > 2 \end{cases}$ is a continuous function.	
Ans	$f(x) = \begin{cases} \frac{x-2}{-(x-2)} + a & ; x < 2 \\ a + b & ; x = 2 \\ \frac{x-2}{(x-2)} + b & ; x > 2 \end{cases} \Rightarrow f(x) = \begin{cases} -1 + a & ; x < 2 \\ a + b & ; x = 2 \\ 1 + b & ; x > 2 \end{cases}$ $\lim_{x \rightarrow 2^-} f(x) = -1 + a, \lim_{x \rightarrow 2^+} f(x) = 1 + b$ and $f(2) = a + b$ as $f$ is continuous at $x = 2 \therefore -1 + a = 1 + b = a + b$ $\Rightarrow a = 1, b = -1$	 1 1 $\frac{1}{2} + \frac{1}{2}$
Q27(a)	Find the intervals in which the function $f(x) = \frac{\log x}{x}$ is strictly increasing or strictly decreasing.	
Ans	$f(x) = \frac{\log x}{x} \Rightarrow f'(x) = \frac{1 - \log x}{x^2}; x > 0$ for strictly increasing/decreasing, put $f'(x) = 0 \Rightarrow x = e$ for strictly increasing, $x \in (0, e)$ and for strictly decreasing $x \in (e, \infty)$	 1  1  $\frac{1}{2} + \frac{1}{2}$

Q27(b)	Find the absolute maximum and absolute minimum values of the function $f$ given by $f(x) = \frac{x}{2} + \frac{2}{x}$ , on the interval $[1, 2]$ .	
Ans	$f(x) = \frac{x}{2} + \frac{2}{x} ; x \in [1, 2]$ $\Rightarrow f'(x) = \frac{1}{2} - \frac{2}{x^2}$ <p>for absolute maximum/ minimum, put <math>f'(x) = 0</math></p> $\Rightarrow x^2 = 4 \Rightarrow x = 2$ <p>Now, <math>f(1) = \frac{5}{2}</math> and <math>f(2) = 2</math></p> <p><math>\therefore</math> absolute maximum value = <math>\frac{5}{2}</math> and absolute minimum value = <math>2</math></p>	<p>1</p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2} + \frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p>
Q28	Find :	
Ans	$\int \frac{x^2 + 1}{(x^2 + 2)(x^2 + 4)} dx$ $I = \int \frac{x^2 + 1}{(x^2 + 2)(x^2 + 4)} dx$ <p>Let <math>x^2 = y</math>, then <math>\frac{x^2 + 1}{(x^2 + 2)(x^2 + 4)} = \frac{y + 1}{(y + 2)(y + 4)}</math></p> <p>Let <math>\frac{y + 1}{(y + 2)(y + 4)} = \frac{A}{y + 2} + \frac{B}{y + 4}</math></p> <p>this gives <math>A = -\frac{1}{2}, B = \frac{3}{2}</math></p> $\therefore I = -\frac{1}{2} \int \frac{1}{x^2 + 2} dx + \frac{3}{2} \int \frac{1}{x^2 + 4} dx$ $\Rightarrow I = -\frac{1}{2\sqrt{2}} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + \frac{3}{4} \tan^{-1}\left(\frac{x}{2}\right) + c$	<p>1</p> <p>1</p> <p>1</p>
Q29(a)	Find :	
	$\int \frac{2 + \sin 2x}{1 + \cos 2x} e^x dx$	

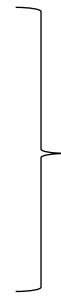
Ans	$I = \int \frac{2 + \sin 2x}{1 + \cos 2x} e^x dx$ $= \int \frac{2 + 2 \sin x \cos x}{2 \cos^2 x} e^x dx$ $= \int (\sec^2 x + \tan x) e^x dx$ $= e^x \cdot \tan x + c$	   1 1 1
OR		
Q29(b)	<p>Evaluate :</p> $\int_0^{\pi/4} \frac{1}{\sin x + \cos x} dx$	
Ans	$I = \int_0^{\pi/4} \frac{1}{\sin x + \cos x} dx$ $= \frac{1}{\sqrt{2}} \int_0^{\pi/4} \frac{1}{\cos \frac{\pi}{4} \sin x + \sin \frac{\pi}{4} \cos x} dx$ $= \frac{1}{\sqrt{2}} \int_0^{\pi/4} \frac{1}{\sin \left( x + \frac{\pi}{4} \right)} dx = \frac{1}{\sqrt{2}} \int_0^{\pi/4} \operatorname{cosec} \left( x + \frac{\pi}{4} \right) dx$ $= \frac{1}{\sqrt{2}} \left[ \log \left  \operatorname{cosec} \left( x + \frac{\pi}{4} \right) - \cot \left( x + \frac{\pi}{4} \right) \right  \right]_0^{\pi/4}$ $= \frac{1}{\sqrt{2}} \log(\sqrt{2} + 1) \text{ or } -\frac{1}{\sqrt{2}} \log(\sqrt{2} - 1)$	          1          1
Q30	<p>Solve the following linear programming problem graphically :</p> <p>Maximise <math>z = 4x + 3y</math>,  subject to the constraints</p> $x + y \leq 800$ $2x + y \leq 1000$ $x \leq 400$ $x, y \geq 0.$	

Ans



Corner Point	Value of $z = 4x + 3y$
$O(0,0)$	0
$A(400,0)$	1600
$B(400,200)$	2200
$C(200,600)$	2600
$D(0,800)$	2400

$z_{\max} = 2600$  when  $x = 200, y = 600$



For correct graph  
 $1\frac{1}{2}$

For correct table  
1

$\frac{1}{2}$

Q31

The chances of P, Q and R getting selected as CEO of a company are in the ratio 4 : 1 : 2 respectively. The probabilities for the company to increase its profits from the previous year under the new CEO, P, Q or R are 0.3, 0.8 and 0.5 respectively. If the company increased the profits from the previous year, find the probability that it is due to the appointment of R as CEO.

Ans	<p>Let <math>E_1</math> : <math>P</math> is appointed as <math>CEO</math>,</p> <p><math>E_2</math> : <math>Q</math> is appointed as <math>CEO</math>,</p> <p><math>E_3</math> : <math>R</math> is appointed as <math>CEO</math></p> <p><math>A</math> : company increase profits from previous year</p> <p>here, <math>P(E_1) = \frac{4}{7}, P(E_2) = \frac{1}{7}, P(E_3) = \frac{2}{7}</math></p> <p><math>P(A E_1) = 0.3, P(A E_2) = 0.8, P(A E_3) = 0.5</math></p> $P(E_3 A) = \frac{P(E_3)P(A E_3)}{P(E_1)P(A E_1) + P(E_2)P(A E_2) + P(E_3)P(A E_3)}$ $= \frac{\frac{2}{7} \times 0.5}{\frac{4}{7} \times 0.3 + \frac{1}{7} \times 0.8 + \frac{2}{7} \times 0.5}$ $= \frac{1}{3}$	<p>1/2</p> <p>1</p> <p>1</p> <p>1/2</p>
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**SECTION D**

Questions no. 32 to 35 are long answer (LA) type questions carrying 5 marks each.

Q32	<p>A relation <math>R</math> on set <math>A = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}</math> be defined as <math>R = \{(x, y) : x + y \text{ is an integer divisible by } 2\}</math>. Show that <math>R</math> is an equivalence relation. Also, write the equivalence class <math>[2]</math>.</p>	
Ans	<p>For reflexive: clearly <math>x + x</math> i.e. <math>2x</math> is integer divisible by 2.</p> <p><math>\Rightarrow (x, x) \in R \Rightarrow R</math> is reflexive.</p> <p>For symmetric: <math>(x, y) \in R \Rightarrow x + y</math> is integer divisible by 2.</p> <p><math>\Rightarrow y + x</math> is integer divisible by 2 <math>\Rightarrow (y, x) \in R</math></p> <p>For transitive: <math>(x, y) \in R \Rightarrow x + y</math> is integer divisible by 2.</p> <p>and <math>(y, z) \in R \Rightarrow y + z</math> is integer divisible by 2.</p> <p>so, <math>(x + z) + 2y</math> is integer divisible by 2.</p> <p><math>\Rightarrow x + z</math> is integer divisible by 2 <math>\Rightarrow (x, z) \in R</math></p> <p>Equivalence class <math>[2] = \{-4, -2, 0, 2, 4\}</math></p>	<p>1</p> <p>1</p> <p>2</p> <p>1</p>
Q33(a)	<p>It is given that function <math>f(x) = x^4 - 62x^2 + ax + 9</math> attains local maximum value at <math>x = 1</math>. Find the value of 'a', hence obtain all other points where the given function <math>f(x)</math> attains local maximum or local minimum values.</p>	

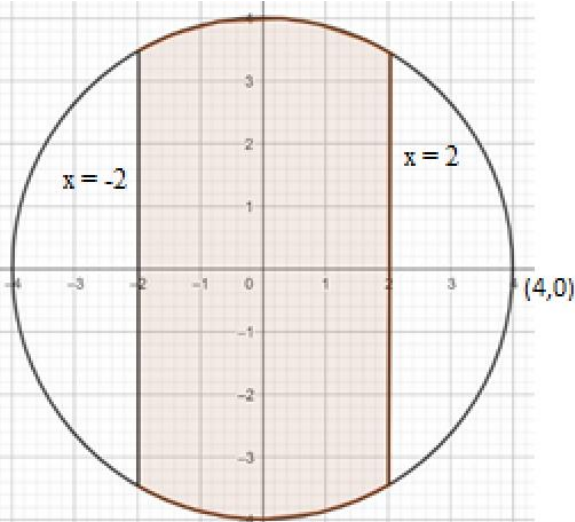
Ans	$f(x) = x^4 - 62x^2 + ax + 9 \Rightarrow f'(x) = 4x^3 - 124x + a$ as at $x = 1$ , $f$ attains local maximum value, $f'(1) = 0 \Rightarrow a = 120$ now, $f'(x) = 4x^3 - 124x + 120 = 4(x-1)(x^2 + x - 30) = 4(x-1)(x-5)(x+6)$ Critical points are $x = -6, 1, 5$ $f''(x) = 12x^2 - 124$ $f''(-6) > 0$ , $f''(1) < 0$ , $f''(5) > 0$ so $f$ attains local maximum value at $x = 1$ and local minimum value at $x = -6, 5$	$\frac{1}{2}$ 1 1 1 $\frac{1}{2}$ 1
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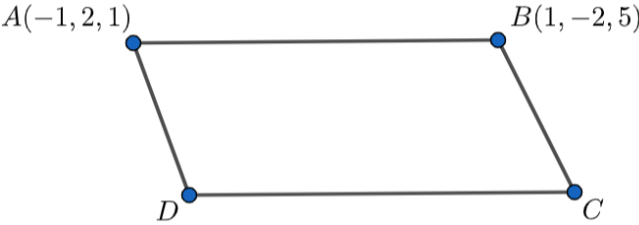
OR

Q33(b)	<p>The perimeter of a rectangular metallic sheet is 300 cm. It is rolled along one of its sides to form a cylinder. Find the dimensions of the rectangular sheet so that volume of cylinder so formed is maximum.</p>	
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Ans	Let length of rectangle be $x$ cm and breadth be $(150 - x)$ cm. Let $r$ be the radius of cylinder $\Rightarrow 2\pi r = x \Rightarrow r = \frac{x}{2\pi}$ $V = \pi r^2 h = \pi \left( \frac{x^2}{4\pi^2} \right) (150 - x) = \frac{75x^2}{2\pi} - \frac{x^3}{4\pi}$ $\frac{dV}{dx} = \frac{150x}{2\pi} - \frac{3x^2}{4\pi}$ $\frac{dV}{dx} = 0 \Rightarrow x = 100$ cm $\left. \frac{d^2V}{dx^2} \right _{x=100 \text{ cm}} = -\frac{75}{\pi} < 0 \Rightarrow V$ is maximum when $x = 100$ cm. Length of rectangle is 100 cm and breadth of rectangle is 50 cm.	1 1 1 1 $\frac{1}{2}$ $\frac{1}{2}$
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Q34	Using integration, find the area of the region enclosed between the circle $x^2 + y^2 = 16$ and the lines $x = -2$ and $x = 2$ .	
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Ans		For correct figure 1 mark
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	$\text{Required area} = 4 \int_0^2 \sqrt{16-x^2} dx$ $= 4 \left[ \frac{x}{2} \sqrt{16-x^2} + 8 \sin^{-1} \left( \frac{x}{4} \right) \right]_0^2$ $= 8\sqrt{3} + \frac{16\pi}{3}$	<p>1</p> <p>2</p> <p>1</p>
Q35(a)	<p>Find the equation of the line passing through the point of intersection of the lines <math>\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}</math> and <math>\frac{x-1}{0} = \frac{y}{-3} = \frac{z-7}{2}</math> and perpendicular to these given lines.</p>	
Ans	$l_1: \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda ; l_2: \frac{x-1}{0} = \frac{y}{-3} = \frac{z-7}{2} = \mu$ <p>any point on <math>l_1</math> is <math>(\lambda, 2\lambda+1, 3\lambda+2)</math> &amp; any point on <math>l_2</math> is <math>(1, -3\mu, 2\mu+7)</math></p> <p>If <math>l_1</math> and <math>l_2</math> intersect,</p> $\lambda = 1, 2\lambda+1 = -3\mu \text{ and } 3\lambda+2 = 2\mu+7 \Rightarrow \lambda = 1 \text{ and } \mu = -1$ <p>Point of intersection of <math>l_1</math> and <math>l_2</math> is <math>(1, 3, 5)</math>.</p> <p>Let d.r.'s of required line be <math>\langle a, b, c \rangle</math>. Then,</p> $a+2b+3c=0 \text{ and } -3b+2c=0 \Rightarrow \frac{a}{13} = \frac{b}{-2} = \frac{c}{-3}$ <p>Required equation of line is <math>\frac{x-1}{13} = \frac{y-3}{-2} = \frac{z-5}{-3}</math></p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>
OR		
Q35(b)	<p>Two vertices of the parallelogram ABCD are given as <math>A(-1, 2, 1)</math> and <math>B(1, -2, 5)</math>. If the equation of the line passing through C and D is <math>\frac{x-4}{1} = \frac{y+7}{-2} = \frac{z-8}{2}</math>, then find the distance between sides AB and CD. Hence, find the area of parallelogram ABCD.</p>	
Ans	 <p>d.r.'s of CD are <math>\langle 1, -2, 2 \rangle</math></p> <p><math>\therefore</math> d.r.'s of AB are <math>\langle 1, -2, 2 \rangle</math></p>	<p><math>\frac{1}{2}</math></p>

	$\therefore \text{Equation of AB is } \frac{x+1}{1} = \frac{y-2}{-2} = \frac{z-1}{2}$ $\therefore \text{Equation of CD is } \frac{x-4}{1} = \frac{y+7}{-2} = \frac{z-8}{2}$ <p>Let <math>\vec{a}_1 = -\hat{i} + 2\hat{j} + \hat{k}</math>, <math>\vec{a}_2 = 4\hat{i} - 7\hat{j} + 8\hat{k}</math> &amp; <math>\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}</math></p> <p>Now, <math>\vec{a}_2 - \vec{a}_1 = 5\hat{i} - 9\hat{j} + 7\hat{k}</math></p> $(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -9 & 7 \\ 1 & -2 & 2 \end{vmatrix} = -4\hat{i} - 3\hat{j} - \hat{k}$ <p>Distance between AB and CD is given by <math>d = \frac{ (\vec{a}_2 - \vec{a}_1) \times \vec{b} }{ \vec{b} }</math></p> $d = \frac{\sqrt{16+9+1}}{\sqrt{1+4+4}} = \frac{\sqrt{26}}{3}$ $CD = \sqrt{2^2 + (-4)^2 + (4)^2} = 6$ <p>Area of parallelogram ABCD = <math>b \times h = 6 \times \frac{\sqrt{26}}{3} = 2\sqrt{26}</math></p>	<p>1/2</p> <p>1</p> <p>1</p> <p>1/2</p> <p>1/2</p> <p>1</p>
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### SECTION E

Questions no. 36 to 38 are case study based questions carrying 4 marks each.

Q36	<p>Self-study helps students to build confidence in learning. It boosts the self-esteem of the learners. Recent surveys suggested that close to 50% learners were self-taught using internet resources and upskilled themselves.</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  <p>SELF-STUDY</p> </div> <div style="text-align: center;">  </div> </div>	
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A student may spend 1 hour to 6 hours in a day in upskilling self. The probability distribution of the number of hours spent by a student is given below :

$$P(X = x) = \begin{cases} kx^2, & \text{for } x = 1, 2, 3 \\ 2kx, & \text{for } x = 4, 5, 6 \\ 0, & \text{otherwise} \end{cases}$$

where x denotes the number of hours.

Based on the above information, answer the following questions :

- (i) Express the probability distribution given above in the form of a probability distribution table. 1
- (ii) Find the value of k. 1
- (iii) (a) Find the mean number of hours spent by the student. 2

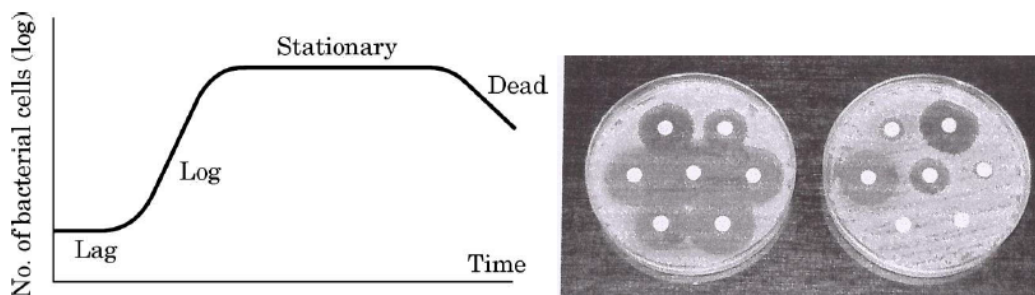
**OR**

- (iii) (b) Find  $P(1 < X < 6)$ . 2

Ans(i)	<table border="1" style="margin: auto; border-collapse: collapse;"> <tr> <td style="padding: 5px;">X</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">2</td> <td style="padding: 5px;">3</td> <td style="padding: 5px;">4</td> <td style="padding: 5px;">5</td> <td style="padding: 5px;">6</td> </tr> <tr> <td style="padding: 5px;">P(X)</td> <td style="padding: 5px;">k</td> <td style="padding: 5px;">4k</td> <td style="padding: 5px;">9k</td> <td style="padding: 5px;">8k</td> <td style="padding: 5px;">10k</td> <td style="padding: 5px;">12k</td> </tr> </table>	X	1	2	3	4	5	6	P(X)	k	4k	9k	8k	10k	12k	1	1
X	1	2	3	4	5	6											
P(X)	k	4k	9k	8k	10k	12k											
Ans(ii)	$k + 4k + 9k + 8k + 10k + 12k = 1$ $\Rightarrow k = \frac{1}{44}$	1															
Ans (iii) (a)	$\text{Mean} = \sum x_i p_i = k + 8k + 27k + 32k + 50k + 72k$ $= 190k$ $= \frac{190}{44} \text{ or } \frac{95}{22}$	1	1														
<b>OR</b>																	
Ans (iii)(b)	$P(1 < X < 6) = 4k + 9k + 8k + 10k$ $= 31k$ $= \frac{31}{44}$	1	1														

Q37

A bacteria sample of certain number of bacteria is observed to grow exponentially in a given amount of time. Using exponential growth model, the rate of growth of this sample of bacteria is calculated.



The differential equation representing the growth of bacteria is given as :

$$\frac{dP}{dt} = kP, \text{ where } P \text{ is the population of bacteria at any time 't'.$$

Based on the above information, answer the following questions :

- (i) Obtain the general solution of the given differential equation and express it as an exponential function of 't'. 2
- (ii) If population of bacteria is 1000 at  $t = 0$ , and 2000 at  $t = 1$ , find the value of  $k$ . 2

Ans(i)	$\frac{dP}{dt} = kP \Rightarrow \int \frac{dP}{P} = \int k dt$ $\Rightarrow \log P = kt + C \text{ or } P = e^{kt+C}$	1  1
Ans(ii)	$\log P = kt + C$ <p>when <math>t = 0, P = 1000 \Rightarrow C = \log 1000</math></p> <p>when <math>t = 1, P = 2000 \Rightarrow \log 2000 = k + \log 1000</math></p> $\Rightarrow k = \log 2$	$\frac{1}{2}$ $\frac{1}{2}$ 1

Q38

A scholarship is a sum of money provided to a student to help him or her pay for education. Some students are granted scholarships based on their academic achievements, while others are rewarded based on their financial needs.



Every year a school offers scholarships to girl children and meritorious achievers based on certain criteria. In the session 2022 – 23, the school offered monthly scholarship of ₹ 3,000 each to some girl students and ₹ 4,000 each to meritorious achievers in academics as well as sports.

In all, 50 students were given the scholarships and monthly expenditure incurred by the school on scholarships was ₹ 1,80,000.

Based on the above information, answer the following questions :

- (i) Express the given information algebraically using matrices. 1
- (ii) Check whether the system of matrix equations so obtained is consistent or not. 1
- (iii) (a) Find the number of scholarships of each kind given by the school, using matrices. 2

**OR**

- (iii) (b) Had the amount of scholarship given to each girl child and meritorious student been interchanged, what would be the monthly expenditure incurred by the school ? 2

Ans(i)	<p>Let No. of girl child scholarships = x</p> <p>No. of meritorious achievers = y</p> <p><math>x + y = 50</math></p> <p><math>3000x + 4000y = 180000</math> or <math>3x + 4y = 180</math></p> $\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 180 \end{bmatrix}$	1
Ans(ii)	$\begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix} = 1 \neq 0$	

	$\therefore$ system is consistent.	1
Ans (iii)(a)	<p>Let <math>A = \begin{bmatrix} 1 &amp; 1 \\ 3 &amp; 4 \end{bmatrix}</math>, <math>X = \begin{bmatrix} x \\ y \end{bmatrix}</math>, <math>B = \begin{bmatrix} 50 \\ 180 \end{bmatrix}</math></p> <p><math>AX = B \Rightarrow X = A^{-1}B</math></p> <p><math>X = \begin{bmatrix} 4 &amp; -1 \\ -3 &amp; 1 \end{bmatrix} \begin{bmatrix} 50 \\ 180 \end{bmatrix} = \begin{bmatrix} 20 \\ 30 \end{bmatrix}</math></p> <p><math>\Rightarrow x = 20, y = 30</math></p>	<p><math>\frac{1}{2}</math></p> <p>1</p> <p><math>\frac{1}{2}</math></p>
OR		
Ans (iii)(b)	<p>Required expenditure = ₹ [30(3000) + 20(4000)]</p> <p style="text-align: center;">= ₹ 1,70,000</p>	<p>1</p> <p>1</p>