## MARKING SCHEME – 65/5/1

| Q.No. | EXPECTED ANSWER / VALUE POINTS | Marks |
|---|---|---|
| | **SECTION-A**<br>**(Question nos. 1 to 18 are Multiple choice Questions carrying 1 mark each)** | |
| **1.** | A function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(x) = x^2 - 4x + 5$ is :<br>(A) injective but not surjective. (B) surjective but not injective.<br>(C) both injective and surjective. (D) neither injective nor surjective. | |
| **Ans** | **(D) neither injective nor surjective** | **1** |
| **2.** | If $A = \begin{bmatrix} a & c & -1 \\ b & 0 & 5 \\ 1 & -5 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then the value of $2a - (b + c)$ is :<br>(A) 0 (B) 1<br>(C) –10 (D) 10 | |
| **Ans** | **(A) 0** | **1** |
| **3.** | If A is a square matrix of order 3 such that the value of $\lvert adj \cdot A \rvert = 8$, then the value of $\lvert A^T \rvert$ is :<br>(A) $\sqrt{2}$ (B) $-\sqrt{2}$<br>(C) 8 (D) $2\sqrt{2}$ | |
| **Ans** | **(D) $2\sqrt{2}$** | **1** |
| **4.** | If inverse of matrix $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is the matrix $\begin{bmatrix} 1 & 3 & 3 \\ 1 & \lambda & 3 \\ 1 & 3 & 4 \end{bmatrix}$, then value of $\lambda$ is :<br>(A) –4 (B) 1<br>(C) 3 (D) 4 | |
| **Ans** | **(D) 4** | **1** |

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| **5.** | If $\begin{bmatrix} x & 2 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ -1 \\ x \end{bmatrix} = \begin{bmatrix} 3 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ x \end{bmatrix}$, then value of $x$ is :<br>(A) –1 (B) 0<br>(C) 1 (D) 2 | |
| **Ans** | **(A) –1** | **1** |
| **6.** | Find the matrix $A^2$, where $A = [a_{ij}]$ is a $2 \times 2$ matrix whose elements are given by $a_{ij}$ = maximum (i, j) – minimum (i, j) :<br>(A) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$<br>(C) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ | |
| **Ans** | **(C)** $\begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$ | **1** |
| **7.** | If $xe^y = 1$, then the value of $\frac{dy}{dx}$ at $x = 1$ is :<br>(A) –1 (B) 1<br>(C) –e (D) $-\frac{1}{e}$ | |
| **Ans** | **(A) –1** | **1** |
| **8.** | Derivative of $e^{\sin^2 x}$ with respect to $\cos x$ is :<br>(A) $\sin x\, e^{\sin^2 x}$ (B) $\cos x\, e^{\sin^2 x}$<br>(C) $-2 \cos x\, e^{\sin^2 x}$ (D) $-2 \sin^2 x \cos x\, e^{\sin^2 x}$ | |
| **Ans** | **(C)** $\mathbf{-2\cos x\, e^{\sin^2 x}}$ | **1** |
| **9.** | The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minima at $x$ equal to :<br>(A) 2 (B) 1<br>(C) 0 (D) –2 | |
| **Ans** | **(A) 2** | **1** |

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| **10.** | Given a curve $y = 7x - x^3$ and $x$ increases at the rate of 2 units per second. The rate at which the slope of the curve is changing, when $x = 5$ is :<br>(A) –60 units/sec (B) 60 units/sec<br>(C) –70 units/sec (D) –140 units/sec | |
| **Ans** | **(A) –60 units/sec** | **1** |
| **11.** | $\int \frac{1}{x(\log x)^2}\,dx$ is equal to :<br>(A) $2\log(\log x) + c$ (B) $-\frac{1}{\log x} + c$<br>(C) $\frac{(\log x)^3}{3} + c$ (D) $\frac{3}{(\log x)^3} + c$ | |
| **Ans** | **(B)** $-\frac{1}{\log x} + c$ | **1** |
| **12.** | The value of $\int_{-1}^{1} x\,\lvert x \rvert\,dx$ is :<br>(A) $\frac{1}{6}$ (B) $\frac{1}{3}$<br>(C) $-\frac{1}{6}$ (D) 0 | |
| **Ans** | **(D) 0** | **1** |
| **13.** | Area of the region bounded by curve $y^2 = 4x$ and the X-axis between $x = 0$ and $x = 1$ is :<br>(A) $\frac{2}{3}$ (B) $\frac{8}{3}$<br>(C) 3 (D) $\frac{4}{3}$ | |
| **Ans** | **(D)** $\frac{4}{3}$ | **1** |
| **14.** | The order of the differential equation $\frac{d^4y}{dx^4} - \sin\left(\frac{d^2y}{dx^2}\right) = 5$ is :<br>(A) 4 (B) 3<br>(C) 2 (D) not defined | |
| **Ans** | **(A) 4** | **1** |

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| **15.** | The position vectors of points P and Q are $\vec{p}$ and $\vec{q}$ respectively. The point R divides line segment PQ in the ratio 3 : 1 and S is the mid-point of line segment PR. The position vector of S is :<br>(A) $\frac{\vec{p}+3\vec{q}}{4}$ (B) $\frac{\vec{p}+3\vec{q}}{8}$<br>(C) $\frac{5\vec{p}+3\vec{q}}{4}$ (D) $\frac{5\vec{p}+3\vec{q}}{8}$ | |
| **Ans** | **(D)** $\frac{5\vec{p}+3\vec{q}}{8}$ | **1** |
| **16.** | The angle which the line $\frac{x}{1}=\frac{y}{-1}=\frac{z}{0}$ makes with the positive direction of Y-axis is :<br>(A) $\frac{5\pi}{6}$ (B) $\frac{3\pi}{4}$<br>(C) $\frac{5\pi}{4}$ (D) $\frac{7\pi}{4}$ | |
| **Ans** | **(B)** $\frac{3\pi}{4}$ | **1** |
| **17.** | The Cartesian equation of the line passing through the point (1, –3, 2) and parallel to the line :<br>$\vec{r}=(2+\lambda)\hat{i}+\lambda\hat{j}+(2\lambda-1)\hat{k}$ is<br>(A) $\frac{x-1}{2}=\frac{y+3}{0}=\frac{z-2}{-1}$ (B) $\frac{x+1}{1}=\frac{y-3}{1}=\frac{z+2}{2}$<br>(C) $\frac{x+1}{2}=\frac{y-3}{0}=\frac{z+2}{-1}$ (D) $\frac{x-1}{1}=\frac{y+3}{1}=\frac{z-2}{2}$ | |
| **Ans** | **(D)** $\frac{x-1}{1}=\frac{y+3}{1}=\frac{z-2}{2}$ | **1** |
| **18.** | If A and B are events such that P(A/B) = P(B/A) $\neq$ 0, then :<br>(A) $A \subset B$, but $A \neq B$ (B) A = B<br>(C) $A \cap B = \phi$ (D) P(A) = P(B) | |
| **Ans** | **(D)** $P(A) = P(B)$ | **1** |

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| **(Question Nos. 19 & 20 are Assertion-Reason based questions of 1 mark each)** | | |
| | **Assertion – Reason Based Questions**<br><br>**Direction :** In questions numbers **19** and **20**, two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the following options :<br><br>(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).<br><br>(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).<br><br>(C) Assertion (A) is true, but Reason (R) is false.<br><br>(D) Assertion (A) is false, but Reason (R) is true. | |
| **19.** | **Assertion (A)** : Domain of $y = \cos^{-1}(x)$ is $[-1, 1]$.<br><br>**Reason (R)** : The range of the principal value branch of $y = \cos^{-1}(x)$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$. | |
| **Ans** | **(C) Assertion (A) is true, but Reason (R) is false** | **1** |
| **20.** | **Assertion (A)** : The vectors<br>$\vec{a} = 6\hat{i} + 2\hat{j} - 8\hat{k}$<br>$\vec{b} = 10\hat{i} - 2\hat{j} - 6\hat{k}$<br>$\vec{c} = 4\hat{i} - 4\hat{j} + 2\hat{k}$<br>represent the sides of a right angled triangle.<br><br>**Reason (R)** : Three non-zero vectors of which none of two are collinear forms a triangle if their resultant is zero vector or sum of any two vectors is equal to the third. | |
| **Ans** | **(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A)** | **1** |
| **SECTION-B**<br>**(Question nos. 21 to 25 are very short Answer type questions carrying 2 marks each)** | | |
| **21.** | Find value of k if<br>$\sin^{-1}\left[k\tan\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right] = \frac{\pi}{3}$. | |

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| **Ans** | $k\tan\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)=\sin\frac{\pi}{3}$ <br> $\Rightarrow k\tan\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}$ <br> $\Rightarrow k\sqrt{3}=\frac{\sqrt{3}}{2}\ \therefore k=\frac{1}{2}$ | $\frac{1}{2}$ <br> $\frac{1}{2}+\frac{1}{2}$ <br> $\frac{1}{2}$ |
| **22.** | (a) Verify whether the function f defined by <br> $f(x)=\begin{cases} x\sin\left(\frac{1}{x}\right), & x\neq 0 \\ 0, & x=0 \end{cases}$ <br> is continuous at $x = 0$ or not. <br> **OR** <br> (b) Check for differentiability of the function f defined by $f(x) = \|x - 5\|$, at the point $x = 5$. | |
| **Ans** | **(a)** $\lim_{x\to 0} f(x)=\lim_{x\to 0} x.\sin\frac{1}{x}=0\times$ **Finite value in** $[-1,1]=0=f(0)$ <br> $\therefore$ **f is a continuous function.** <br> **Or** <br> **(b)** $\text{LHD}=\lim_{x\to 5^-}\frac{\|x-5\|-0}{x-5}=\lim_{x\to 5^-}\frac{-(x-5)}{x-5}=-1$ <br> $\text{RHD}=\lim_{x\to 5^+}\frac{\|x-5\|-0}{x-5}=\lim_{x\to 5^+}\frac{(x-5)}{x-5}=1$ <br> **LHD ≠ RHD,** $\therefore$ **f is not differentiable at** $x=5$ | $1\frac{1}{2}$ <br> $\frac{1}{2}$ <br> <br> 1 <br> $\frac{1}{2}$ <br> $\frac{1}{2}$ |
| **23.** | The area of the circle is increasing at a uniform rate of 2 $cm^2$/sec. How fast is the circumference of the circle increasing when the radius r = 5 cm ? | |
| **Ans** | **Let** $C=2\pi r$**, be the circumference of the circle,** <br> $\frac{d(\pi r^2)}{dt}=2\Rightarrow\frac{dr}{dt}=\frac{1}{\pi r}$ **cm / sec** <br> $\frac{dC}{dt}=2\pi\frac{dr}{dt}=\frac{2}{r}=\frac{2}{5}$ **cm/sec at** $r=5$**cm** | 1 <br> 1 |

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| **24.** | (a) Find : $\int \cos^3 x\, e^{\log \sin x}\, dx$<br>**OR**<br>(b) Find : $\int \frac{1}{5+4x-x^2}\, dx$ | |
| **Ans** | **(a)** $\int \cos^3 x\, e^{\log \sin x} dx = \int \cos^3 x \cdot \sin x\, dx$, **Assuming** $\cos x = t$ **and** $\sin x\, dx = -\, dt$<br>$= -\int t^3 dt$<br>$= -\frac{t^4}{4} + C = -\frac{\cos^4 x}{4} + C$<br>**Or**<br>**(b)** $\int \frac{1}{5+4x-x^2} dx = \int \frac{1}{3^2 - (x-2)^2} dx$<br>$= \frac{1}{6} \log \left\| \frac{1+x}{5-x} \right\| + C$ | 1<br>½<br>½<br><br>1<br>1 |
| **25.** | Find the vector equation of the line passing through the point (2, 3, –5) and making equal angles with the co-ordinate axes. | |
| **Ans** | **(a)** $\cos\alpha = \cos\beta = \cos\gamma = l \Rightarrow l^2 + l^2 + l^2 = 1 \Rightarrow 3l^2 = 1,\ \therefore l = 1/\sqrt{3}$<br>**Direction cosines of the line are** $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ $\Rightarrow$ **the direction ratios are 1,1,1**<br>$\therefore$ **Vector equation of the line is:** $\vec{r} = 2\hat{i} + 3\hat{j} - 5\hat{k} + \lambda(\hat{i} + \hat{j} + \hat{k})$ | 1<br>½<br>½ |

**SECTION-C**

**(Question nos. 26 to 31 are short Answer type questions carrying 3 marks each)**

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| **26.** | (a) Find $\frac{dy}{dx}$, if $(\cos x)^y = (\cos y)^x$.<br>**OR**<br>(b) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$. | |
| **Ans** | **(a) Taking 'log' on both sides of** $(\cos x)^y = (\cos y)^x$**, we get**<br>$y \log \cos x = x \log \cos y$ | <br>1 |

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| | $\Rightarrow \frac{dy}{dx}\log\cos x + y(-\tan x) = \log\cos y + x(-\tan y)\frac{dy}{dx}$ | $1\frac{1}{2}$ |
| | $\Rightarrow \frac{dy}{dx} = \frac{\log\cos y + y\tan x}{\log\cos x + x\tan y}$ | $\frac{1}{2}$ |
| | **Or** | |
| | **(b) Let** $x = \sin A, y = \sin B \therefore A = \sin^{-1}x,\ B = \sin^{-1}y$ | $\frac{1}{2}$ |
| | $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ | |
| | $\Rightarrow \cos A + \cos B = a(\sin A - \sin B)$ | |
| | $\Rightarrow 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) = 2a\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$ | 1 |
| | $\Rightarrow \cot\left(\frac{A-B}{2}\right) = a \Rightarrow A - B = 2\cot^{-1}a$ , $\therefore \sin^{-1}x - \sin^{-1}y = 2\cot^{-1}a$ , **differentiating with respect to 'x'.** | $\frac{1}{2}$ |
| | $\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$ | 1 |
| **27.** | If $x = a\sin^3\theta$, $y = b\cos^3\theta$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$. | |
| **Ans** | $\frac{dx}{d\theta} = 3a\sin^2\theta\ \cos\theta,\ \frac{dy}{d\theta} = -3b\cos^2\theta\ \sin\theta$ | 1 |
| | $\Rightarrow \frac{dy}{dx} = \frac{-3b\cos^2\theta\ \sin\theta}{3a\sin^2\theta\ \cos\theta} = -\frac{b}{a}\cot\theta$ | $\frac{1}{2}$ |
| | $\Rightarrow \frac{d^2y}{dx^2} = \frac{b}{a}\operatorname{cosec}^2\theta\ \frac{d\theta}{dx} = \frac{b}{a}\ \operatorname{cosec}^2\theta \cdot \frac{1}{3a\sin^2\theta\ \cos\theta} = \frac{b}{3a^2}\sec\theta\ \operatorname{cosec}^4\theta$ | 1 |
| | $\Rightarrow \left.\frac{d^2y}{dx^2}\right]_{\theta=\frac{\pi}{4}} = \frac{4\sqrt{2}b}{3a^2}$ | $\frac{1}{2}$ |
| **28.** | (a) Evaluate : $\int_0^{\pi}\frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}}dx$ <br> **OR** <br> (b) Find : $\int\frac{2x+1}{(x+1)^2\ (x-1)}dx$ | |
| **Ans** | **(a) Let** $I = \int_0^{\pi}\frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}}dx$ --------- (i) | |
| | $\Rightarrow I = \int_0^{\pi}\frac{e^{\cos(\pi-x)}}{e^{\cos(\pi-x)} + e^{-\cos(\pi-x)}}dx = \int_0^{\pi}\frac{e^{-\cos x}}{e^{-\cos x} + e^{\cos x}}dx$ --------- (ii) | $1\frac{1}{2}$ |
| | **Adding (i) and (ii), we get** | |
| | $2I = \int_0^{\pi}dx = x]_0^{\pi} = \pi,\ \therefore I = \frac{\pi}{2}$ | $1\frac{1}{2}$ |
| | **Or** | |

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| | (b) $\int \frac{2x+1}{(x+1)^2(x-1)}dx = -\frac{3}{4}\int\frac{1}{x+1}\,dx + \frac{1}{2}\int\frac{1}{(x+1)^2}\,dx + \frac{3}{4}\int\frac{1}{x-1}\,dx$<br>$= -\frac{3}{4}\log\lvert x+1\rvert - \frac{1}{2(x+1)} + \frac{3}{4}\log\lvert x-1\rvert + C$<br>or, $= \frac{3}{4}\log\left\lvert\frac{x-1}{x+1}\right\rvert - \frac{1}{2(x+1)} + C$ | $1\frac{1}{2}$<br>$1\frac{1}{2}$ |
| **29.** | (a) Find the particular solution of the differential equation<br>$\frac{dy}{dx} - 2xy = 3x^2 e^{x^2}$ ; y(0) = 5.<br>**OR**<br>(b) Solve the following differential equation :<br>$x^2\,dy + y(x+y)\,dx = 0$ | |
| **Ans** | **(a) Given differential equation is a linear order differential equation with:**<br>$P = -2x,\ Q = 3x^2e^{x^2}$<br>**Integrating Factor** $= e^{\int -2x dx} = e^{-x^2}$<br>**The general solution is:** $y\cdot e^{-x^2} = \int e^{-x^2}\cdot 3x^2e^{x^2}dx + C \Rightarrow y\cdot e^{-x^2} = x^3 + C$<br>**Putting** $x=0, y=5$, **we get,** $C=5$<br>$\therefore$ **The Particular solution is:** $y\cdot e^{-x^2} = x^3+5$ or $y = (x^3+5)e^{x^2}$<br>**Or**<br>(b) $x^2dy + y(x+y)dx = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x} - \left(\frac{y}{x}\right)^2$<br>**Putting** $\frac{y}{x} = v \Rightarrow y = vx,\ \frac{dy}{dx} = v + x\frac{dv}{dx}$<br>$v + x\frac{dv}{dx} = -v - v^2$,<br>**separating the variable and integrating**<br>$\int\frac{1}{v^2+2v}dv = -\int\frac{1}{x}dx$<br>$\Rightarrow \int\frac{1}{(v+1)^2-1}dv = -\int\frac{1}{x}dx$<br>$\Rightarrow \frac{1}{2}\log\left\lvert\frac{v}{v+2}\right\rvert = \log\left\lvert\frac{C}{x}\right\rvert$ | <br>$\frac{1}{2}$<br>1<br>1<br><br>$\frac{1}{2}$<br><br>$\frac{1}{2}$<br>$\frac{1}{2}$<br><br><br>$\frac{1}{2}$<br>$\frac{1}{2}$<br>$\frac{1}{2}$ |

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| | **The solution of the differential equation is,** $\left\|\frac{y}{y+2x}\right\| = \frac{C^2}{x^2}$ **or** $x^2y = k(y+2x)$ | ½ |
| **30.** | Find a vector of magnitude 4 units perpendicular to each of the vectors $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + \hat{j} - \hat{k}$ and hence verify your answer. | |
| **Ans** | **Let** $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + \hat{j} - \hat{k}$ **and** $\vec{c}$ **be the vector perpendicular to both** $\vec{a}\,\&\,\vec{b}$ <br> **then,** $\vec{c} = \vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = 3\hat{j} + 3\hat{k}$ <br> **Let** $\vec{d}$ **the vector perpendicular to both the vectors** $\vec{a}\,\&\,\vec{b}$ **and having magnitude 4,** <br> $\vec{d} = 4\hat{c} = 2\sqrt{2}\hat{j} + 2\sqrt{2}\hat{k}$ **(or** $-2\sqrt{2}\hat{j} - 2\sqrt{2}\hat{k}$**)** <br> **Verification:** $\|\vec{d}\| = \sqrt{8+8} = 4$, $\vec{d}\cdot\vec{a} = \vec{d}\cdot\vec{b} = 0 \Rightarrow \vec{d}\perp\vec{a}$ **and** $\vec{d}\perp\vec{b}$ | <br><br>1<br><br>1<br><br>1 |
| **31.** | The random variable X has the following probability distribution where a and b are some constants : <br><br> X: 1, 2, 3, 4, 5 <br> P(X): 0.2, a, a, 0.2, b <br><br> If the mean E(X) = 3, then find values of a and b and hence determine $P(X \geq 3)$. | |
| **Ans** | $E(X) = 0.2 + 2a + 3a + 0.8 + 5b = 5a + 5b + 1$ <br> $\sum p_i = 1 \Rightarrow 2a + b = 0.6$ <br> $E(X) = 3 \Rightarrow 5a + 5b = 2$ **and,** <br> **solving the two equations, we get,** $a = \frac{1}{5}, b = \frac{1}{5}$ <br> $P(X \geq 3) = 1 - [P(X=1) + P(X=2)] = 1 - [0.2 + a] = 1 - \frac{2}{5} = \frac{3}{5}$ | ½<br>½<br><br>1<br>1 |

## SECTION-D

**(Question nos. 32 to 35 are Long Answer type questions carrying 5 marks each)**

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| **32.** | (a) If $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 0 & -3 \\ 1 & 2 & 0 \end{bmatrix}$, then find $A^{-1}$ and hence solve the following system of equations : <br> $x + 2y - 3z = 1$ <br> $2x - 3z = 2$ <br> $x + 2y = 3$ <br> **OR** | |

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| | (b) Find the product of the matrices $\begin{bmatrix}1&2&-3\\2&3&2\\3&-3&-4\end{bmatrix}\begin{bmatrix}-6&17&13\\14&5&-8\\-15&9&-1\end{bmatrix}$ and hence solve the system of linear equations : <br> $x+2y-3z=-4$ <br> $2x+3y+2z=2$ <br> $3x-3y-4z=11$ | |
| **Ans** | **(a)** $\lvert A\rvert=1(6)-2(3)-3(4)=-12\neq 0$, $\therefore$ $A^{-1}$ exist | 1 |
| | $\text{adj}A=\begin{bmatrix}6&-6&-6\\-3&3&-3\\4&0&-4\end{bmatrix}$ | 2 |
| | $\therefore A^{-1}=-\frac{1}{12}\begin{bmatrix}6&-6&-6\\-3&3&-3\\4&0&-4\end{bmatrix}$ | ½ |
| | **The given system of equations can be written as** $AX=B$, $X=\begin{bmatrix}x\\y\\z\end{bmatrix}, B=\begin{bmatrix}1\\2\\3\end{bmatrix}$ | |
| | $X=A^{-1}B\Rightarrow\begin{bmatrix}x\\y\\z\end{bmatrix}=-\frac{1}{12}\begin{bmatrix}6&-6&-6\\-3&3&-3\\4&0&-4\end{bmatrix}\begin{bmatrix}1\\2\\3\end{bmatrix}=\begin{bmatrix}2\\1/2\\2/3\end{bmatrix}$ | 1½ |
| | $\therefore$ **The solution of the given system of equations is:** $x=2, y=\frac{1}{2}, z=\frac{2}{3}$ <br> **Or** | |
| | **(b)** $\begin{bmatrix}1&2&-3\\2&3&2\\3&-3&-4\end{bmatrix}\begin{bmatrix}-6&17&13\\14&5&-8\\-15&9&-1\end{bmatrix}=\begin{bmatrix}67&0&0\\0&67&0\\0&0&67\end{bmatrix}$ | 2½ |
| | $\Rightarrow\begin{bmatrix}1&2&-3\\2&3&2\\3&-3&-4\end{bmatrix}^{-1}=\frac{1}{67}\begin{bmatrix}-6&17&13\\14&5&-8\\-15&9&-1\end{bmatrix}$ | 1 |
| | **Solution of the system of equations is given by:** <br> $\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1&2&-3\\2&3&2\\3&-3&-4\end{bmatrix}^{-1}\begin{bmatrix}-4\\2\\11\end{bmatrix}=\frac{1}{67}\begin{bmatrix}-6&17&13\\14&5&-8\\-15&9&-1\end{bmatrix}\begin{bmatrix}-4\\2\\11\end{bmatrix}=\begin{bmatrix}3\\-2\\1\end{bmatrix}$, <br> $\therefore\ x=3, y=-2, z=1$ | 1½ |

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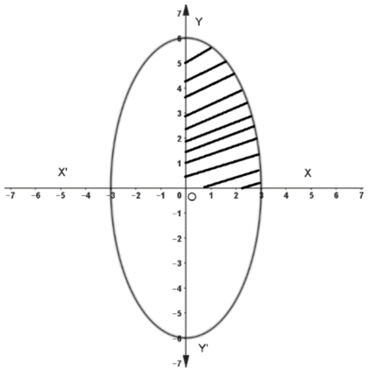
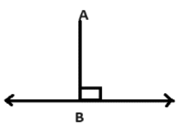
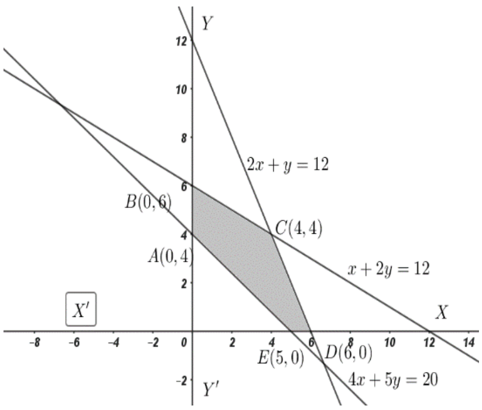
|---|---|---|
| **33.** | Find the area of the region bounded by the curve $4x^2 + y^2 = 36$ using integration. | |
| **Ans** | **The given equation can be written as:** $\frac{x^2}{9}+\frac{y^2}{36}=1$, **which is an ellipse.**<br><br>**Correct Graph**<br>**Area of the region bounded by the curve**<br>$=4\times\frac{6}{3}\int_0^3\sqrt{9-x^2}\,dx$<br>$=8\left[\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}\sin^{-1}\frac{x}{3}\right]_0^3$<br>$=18\pi$ | <br><br>1<br><br>$1\frac{1}{2}$<br>$1\frac{1}{2}$<br>1 |
| **34.** | (a) Find the co-ordinates of the foot of the perpendicular drawn from the point (2, 3, –8) to the line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$.<br>Also, find the perpendicular distance of the given point from the line.<br>**OR**<br>(b) Find the shortest distance between the lines $L_1$ & $L_2$ given below :<br>$L_1$ : The line passing through (2, –1, 1) and parallel to $\frac{x}{1}=\frac{y}{1}=\frac{z}{3}$<br>$L_2$ : $\vec{r}=\hat{i}+(2\mu+1)\hat{j}-(\mu+2)\hat{k}$. | |
| **Ans** | **(a) The standard form of the equation of the line is** $\frac{x-4}{-2}=\frac{y}{6}=\frac{z-1}{-3}$<br><br>**Let foot of the perpendicular from the point** $A(2,3,-8)$ **to the given line be** $B(-2\lambda+4, 6\lambda, -3\lambda+1)$<br>**D-ratios of AB is:** $-2\lambda+2, 6\lambda-3, -3\lambda+9$<br>**As AB is perpendicular to the given line:** $-2(-2\lambda+2)+6(6\lambda-3)-3(-3\lambda+9)=0$<br>$\Rightarrow\lambda=1$<br>$\therefore$ **Foot of the perpendicular is:** $B(2,6,-2)$<br>**Perpendicular distance** $=AB=3\sqrt{5}$<br><br>**Or** | $\frac{1}{2}$<br><br>$\frac{1}{2}$<br>1<br><br>1<br>1<br>1 |

| | | |
|---|---|---|
| | (b) Equation of $L_1$: $\vec{r} = 2\hat{i} - \hat{j} + \hat{k} + \lambda(\hat{i} + \hat{j} + 3\hat{k})$ | 1 |
| | Equation of $L_2$: $\vec{r} = (\hat{i} + \hat{j} - 2\hat{k}) + \mu(2\hat{j} - \hat{k})$ | ½ |
| | Taking $\vec{a}_1 = 2\hat{i} - \hat{j} + \hat{k},\ \vec{b}_1 = \hat{i} + \hat{j} + 3\hat{k}$; $\vec{a}_2 = \hat{i} + \hat{j} - 2\hat{k},\ \vec{b}_2 = 2\hat{j} - \hat{k}$ | ½ |
| | $\vec{a}_2 - \vec{a}_1 = -\hat{i} + 2\hat{j} - 3\hat{k},\ \vec{b}_1 \times \vec{b}_2 = -7\hat{i} + \hat{j} + 2\hat{k}$ | ½+1 |
| | Shortest Distance $= \dfrac{\left|(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)\right|}{\left|\vec{b}_1 \times \vec{b}_2\right|} = \dfrac{1}{\sqrt{6}}$ | 1½ |
| **35.** | Solve the following L.P.P. graphically :<br>Maximise Z = $60x + 40y$<br>Subject to $x + 2y \le 12$<br>$2x + y \le 12$<br>$4x + 5y \ge 20$<br>$x, y \ge 0$ | |
| **Ans** | <br>**Correct Graph** | 3½ |



| Corner Points | Value of $Z = 60x + 40y$ |
|---|---|
| $A(0,4)$ | $Z = 160$ |
| $B(0,6)$ | $Z = 240$ |
| $C(4,4)$ | $Z = 400$ |
| $D(6,0)$ | $Z = 360$ |
| $E(5,0)$ | $Z = 300$ |

(1 mark)

**Max(Z) = 400 at x = 4, y = 4** ½

**SECTION-E**

**(Question nos. 36 to 38 are source based/case based/passage based/integrated units of assessment questions carrying 4 marks each)**

**36.** (a) Students of a school are taken to a railway museum to learn about railways heritage and its history.

An exhibit in the museum depicted many rail lines on the track near the railway station. Let L be the set of all rail lines on the railway track and R be the relation on L defined by

$R = \{(l_1, l_2) : l_1 \text{ is parallel to } l_2\}$

On the basis of the above information, answer the following questions :

(i) Find whether the relation R is symmetric or not.

(ii) Find whether the relation R is transitive or not.

(iii) If one of the rail lines on the railway track is represented by the equation $y = 3x + 2$, then find the set of rail lines in R related to it.

**OR**

(b) Let S be the relation defined by $S = \{(l_1, l_2) : l_1 \text{ is perpendicular to } l_2\}$ check whether the relation S is symmetric and transitive.

**Ans**

**(a) (i) Let $(l_1, l_2) \in R \Rightarrow l_1 \parallel l_2 \Rightarrow l_2 \parallel l_1 \Rightarrow (l_2, l_1) \in R$, $\therefore$ R is a symmetric relation** — 1

**(ii) Let $(l_1, l_2), (l_2, l_3) \in R \Rightarrow l_1 \parallel l_2$, $l_2 \parallel l_3 \Rightarrow l_1 \parallel l_3 \Rightarrow (l_1, l_3) \in R$, $\therefore$ R is a transitive relation** — 1

**(iii) The set is $\{l :\ l \text{ is a line of type } y = 3x + c, c \in R\}$** — 2

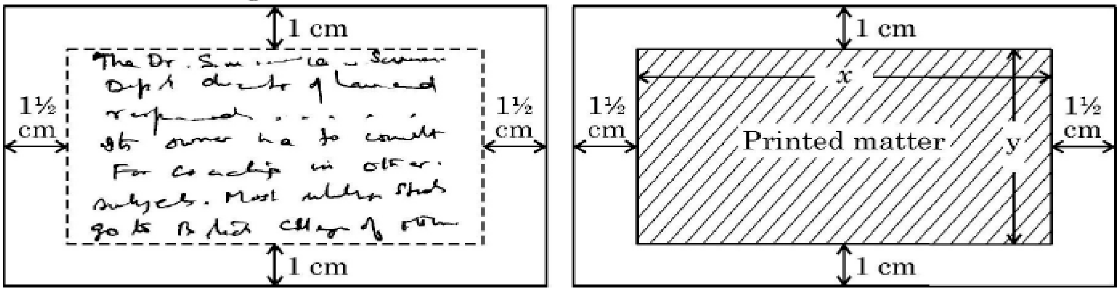
**Or**

**(b) Let $(l_1, l_2) \in R \Rightarrow l_1 \perp l_2 \Rightarrow l_2 \perp l_1 \Rightarrow (l_2, l_1) \in R$, $\therefore$ R is a symmetric relation** — 2

**Let $(l_1, l_2), (l_2, l_3) \in R \Rightarrow l_1 \perp l_2, l_2 \perp l_3 \Rightarrow l_1 \parallel l_3 \Rightarrow (l_1, l_3) \notin R$, $\therefore$ R is not a transitive relation** — 2

**** Due to printing error Part (a) or Part(b), both parts be taken as independent questions of 4 marks each**

**37.** A rectangular visiting card is to contain 24 sq.cm. of printed matter. The margins at the top and bottom of the card are to be 1 cm and the margins on the left and right are to be 1½ cm as shown below :




On the basis of the above information, answer the following questions :

(i) Write the expression for the area of the visiting card in terms of $x$.

(ii) Obtain the dimensions of the card of minimum area.

**Ans**

**(i) Let $A(x)$ be the area of the visiting card then,**

$$\text{As } xy = 24,\ A(x) = (x+3)(y+2) = 2x + 3y + xy + 6 = 2x + \frac{72}{x} + 30$$ **2**

**(ii)** $A'(x) = 2 - \frac{72}{x^2}$ **and** $A''(x) = \frac{144}{x^3}$, **1**

**solving** $A'(x) = 0 \Rightarrow x = 6$ **is the critical point.** **½**

$A''(6) = \frac{144}{6} > 0$, $\therefore$ **Area of the card is minimum at** $x = 6, y = 4$

**The dimension of the card with minimum area is** $\text{Length} = 9$ **cm**, $\text{Breadth} = 6$ **cm** **½**

**38.** A departmental store sends bills to charge its customers once a month. Past experience shows that 70% of its customers pay their first month bill in time. The store also found that the customer who pays the bill in time has the probability of 0.8 of paying in time next month and the customer who doesn't pay in time has the probability of 0.4 of paying in time the next month.

Based on the above information, answer the following questions :

(i) Let $E_1$ and $E_2$ respectively denote the event of customer paying or not paying the first month bill in time.
Find $P(E_1)$, $P(E_2)$.

(ii) Let A denotes the event of customer paying second month's bill in time, then find $P(A|E_1)$ and $P(A|E_2)$.

(iii) Find the probability of customer paying second month's bill in time.

**OR**

(iii) Find the probability of customer paying first month's bill in time if it is found that customer has paid the second month's bill in time.

| | | |
|---|---|---|
| **Ans** | **(i)** $P(E_1) = \frac{7}{10} = 0.7,\ P(E_2) = \frac{3}{10} = 0.3$ | 1 |
| | **(ii)** $P(A \mid E_1) = 0.8, P(A \mid E_2) = 0.4$ | 1 |
| | **(iii)** $P(A) = P(E_1) \cdot P(A \mid E_1) + P(E_2) \cdot P(A \mid E_2) = 0.7 \times 0.8 + 0.3 \times 0.4 = 0.68$ **or** $\frac{17}{25}$ | 2 |
| | **Or** | |
| | **(iii)** $P(A) = \frac{P(E_1) \cdot P(A \mid E_1)}{P(E_1) \cdot P(A \mid E_1) + P(E_2) \cdot P(A \mid E_2)} = \frac{14}{17}$ | 2 |