Q. No.	EXPECTED ANSWER / VALUE POINTS	Marks
	SECTION - A Questions no. 1 to 18 are multiple choice questions (MCQs) of **1 mark each**.	
Q1.	If the matrix $A = [a_{ij}]_{2\times2}$ is such that $a_{ij} = \begin{cases} 1, & \text{if } i \neq j \\ 0, & \text{if } i = j \end{cases}$, then $A + A^2$ is equal to : (A) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	
Ans	(C) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$	1
Q2.	The domain of $f(x) = \cos^{-1}(2x)$ is : (A) $[-1, 1]$ (B) $\left[0, \frac{1}{2}\right]$ (C) $[-2, 2]$ (D) $\left[-\frac{1}{2}, \frac{1}{2}\right]$	
Ans	(D) $\left[-\frac{1}{2}, \frac{1}{2}\right]$	1
Q3.	The value of the determinant $\begin{vmatrix} \cos 75^\circ & \sin 75^\circ \\ \sin 15^\circ & \cos 15^\circ \end{vmatrix}$ is : (A) 1 (B) zero (C) $\frac{1}{2}$ (D) $\frac{\sqrt{3}}{2}$	
Ans	(B) zero	1

Q4.	For a non-singular matrix X, if $X^2 = I$, then X^{-1} is equal to : (A) X (B) X^2 (C) I (D) O	
Ans	(A) X	1
Q5.	The cofactor of the element a_{32} in the determinant $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{vmatrix}$ is : (A) ± 5 (B) -5 (C) 5 (D) 0	
Ans	(C) 5	1
Q6.	If A is an identity matrix of order n, then A (Adj A) is a/an : (A) identity matrix (B) row matrix (C) zero matrix (D) skew symmetric matrix	
Ans	(A) identity matrix	1
Q7.	If $x = t^3$ and $y = t^2$, then $\frac{d^2y}{dx^2}$ at $t = 1$ is : (A) $\frac{3}{2}$ (B) $-\frac{2}{9}$ (C) $-\frac{3}{2}$ (D) $-\frac{2}{3}$	
Ans	(B) $-\frac{2}{9}$	1

Q8.	The area bounded by the parabola $x^2 = y$ and the line $y = 1$ is : (A) $\frac{2}{3}$ sq unit (B) $\frac{1}{3}$ sq unit (C) $\frac{4}{3}$ sq units (D) 2 sq units	
Ans	(C) $\frac{4}{3}$ sq units	1
Q9.	If the rate of change of volume of a sphere is twice the rate of change of its radius, then the surface area of the sphere is : (A) 1 sq unit (B) 2 sq units (C) 3 sq units (D) 4 sq units	
Ans	(B) 2 sq units	1
Q10.	$\int \frac{3\cos\sqrt{x}}{\sqrt{x}}\,dx$ is equal to : (A) $-6\sin\sqrt{x} + C$ (B) $-6\cos\sqrt{x} + C$ (C) $6\cos\sqrt{x} + C$ (D) $6\sin\sqrt{x} + C$	
Ans	(D) $6\sin\sqrt{x} + C$	1

Q11.	If $\frac{d}{dx}f(x) = 3x^2 - \frac{3}{x^4}$ such that f(1) = 0, then f(x) is : (A) $6x + \frac{12}{x^5}$ (B) $x^4 - \frac{1}{x^3} + 2$ (C) $x^3 + \frac{1}{x^3} - 2$ (D) $x^3 + \frac{1}{x^3} + 2$	
Ans	(C) $x^3 + \frac{1}{x^3} - 2$	1
Q12.	In an LPP, corner points of the feasible region determined by the system of linear constraints are (1, 1), (3, 0) and (0, 3). If Z = ax + by, where a, b > 0 is to be minimized, the condition on a and b, so that the minimum of Z occurs at (3, 0) and (1, 1), will be : (A) a = 2b (B) $a = \frac{b}{2}$ (C) a = 3b (D) a = b	
Ans	(B) $a = \frac{b}{2}$	1
Q13.	The maximum value of Z = 3x + 4y subject to the constraints $x + y \leq 1$, $x, y \geq 0$ is : (A) 3 (B) 4 (C) 7 (D) 0	
Ans	(B) 4	1

Q14.	The general solution of the differential equation $\frac{dy}{dx} = 2x \cdot e^{x^2+y}$ is : (A) $e^{x^2+y} = C$ (B) $e^{x^2} + e^{-y} = C$ (C) $e^{x^2} = e^{y} + C$ (D) $e^{x^2-y} = C$	
Ans	(B) $e^{x^2} + e^{-y} = C$	1
Q15.	If 'm' and 'n' are the degree and order respectively of the differential equation $1 + \left(\frac{dy}{dx}\right)^3 = \frac{d^2y}{dx^2}$, then the value of (m + n) is : (A) 4 (B) 3 (C) 2 (D) 5	
Ans	(B) 3	1
Q16.	If $\|\vec{a}\| = 1$, $\|\vec{b}\| = 2$ and $\vec{a} \cdot \vec{b} = 2$, then the value of $\|\vec{a} + \vec{b}\|$ is : (A) 9 (B) 3 (C) – 3 (D) 2	
Ans	(B) 3	1
Q17.	Two vectors $\vec{a}$ and $\vec{b}$ are such that $\|\vec{a} \times \vec{b}\| = \vec{a} \cdot \vec{b}$. The angle between the two vectors is : (A) 30° (B) 60° (C) 45° (D) 90°	
Ans	(C) 45°	1

Q18.	A coin is tossed three times. The probability of getting at least two heads is : (A) $\frac{1}{2}$ (B) $\frac{3}{8}$ (C) $\frac{1}{8}$ (D) $\frac{1}{4}$	
Ans	(A) $\frac{1}{2}$	1

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

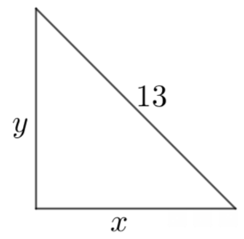
(D) Assertion (A) is false, but Reason (R) is true.

Q19.	Consider the function $f : R \longrightarrow R$, defined as $f(x) = x^3$. *Assertion (A) :* f(x) is a one-one function. *Reason (R) :* f(x) is a one-one function, if co-domain = range.	
Ans	(C) Assertion (A) is true, but Reason (R) is false.	1
Q20.	*Assertion (A) :* $f(x) = [x]$, $x \in \mathbb{R}$, the greatest integer function is not differentiable at x = 2. *Reason (R) :* The greatest integer function is not continuous at any integral value.	
Ans	(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).	1

SECTION B

This section comprises very short answer (VSA) type questions of **2 marks each**.

Q21.	(a) Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$. **OR** (b) Prove that : $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x \in [0, 1]$	
Ans(a)	$\cos^{-1}\left(-\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$ $= \left(\pi - \frac{\pi}{3}\right) + 2\left(\frac{\pi}{6}\right)$ $= \pi$	1 + ½ ½
	OR	
Ans(b)	Put $x = \tan^2\theta \Rightarrow \theta = \tan^{-1}\sqrt{x}$ RHS $= \frac{1}{2}\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$ $= \frac{1}{2}\cos^{-1}(\cos 2\theta)$ $= \frac{1}{2}(2\theta)$ $= \theta = \tan^{-1}\sqrt{x} =$ LHS	½ 1 ½
Q22.	If $e^y(x+1) = 1$, prove that $\frac{dy}{dx} = -e^y$.	
Ans	$e^y(x+1) = 1 \Rightarrow e^y = \frac{1}{x+1}$ $\Rightarrow y = -\log(x+1)$ $\Rightarrow \frac{dy}{dx} = -\frac{1}{x+1}$ $= -e^y \quad \left[\because \frac{1}{x+1} = e^y\right]$	½ 1 ½
Q23.	A ladder 13 m long is leaning against the wall. The bottom of the ladder is pulled along the ground away from the wall at the rate of 2 m/s. How fast is the height on the wall decreasing when the foot of the ladder is 12 m away from the wall ?	

Ans	$x^2 + y^2 = 169$ Differentiate both sides w.r.t. t $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$ $\Rightarrow 12(2) + 5\left(\frac{dy}{dt}\right) = 0 [\because \text{ when } x = 12m, y = 5m]$ $\Rightarrow \frac{dy}{dt} = -\frac{24}{5}$ Hence, the height decreases at the rate of $\frac{24}{5}$ m/s  	½ 1 ½
Q24.	(a) Find the value of λ, if the points (–1, –1, 2), (2, 8, λ) and (3, 11, 6) are collinear. **OR** (b) $\vec{a}$ and $\vec{b}$ are two co-initial vectors forming the adjacent sides of a parallelogram such that $\|\vec{a}\| = 10$, $\|\vec{b}\| = 2$ and $\vec{a}\cdot\vec{b} = 12$. Find the area of the parallelogram.	
Ans(a)	$A(-1,-1,2), B(2,8,\lambda), C(3,11,6)$ $\overrightarrow{AB} = 3\hat{\imath} + 9\hat{\jmath} + (\lambda - 2)\hat{k}$ and $\overrightarrow{BC} = \hat{\imath} + 3\hat{\jmath} + (6 - \lambda)\hat{k}$ Since A, B and C are collinear, $\frac{3}{1} = \frac{9}{3} = \frac{\lambda - 2}{6 - \lambda}$ $\Rightarrow \lambda = 5$	1 ½ ½
	OR	
Ans(b)	Let θ is the angle between $\vec{a}$ and $\vec{b}$. $\vec{a}.\vec{b} = 12 \Rightarrow \|\vec{a}\|\|\vec{b}\| \cos\theta = 12$ $\Rightarrow (10)(2)\cos\theta = 12 \Rightarrow \cos\theta = \frac{3}{5}$ $\therefore \sin\theta = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \frac{4}{5}$ Now, area of parallelogram $= \|\vec{a} \times \vec{b}\| = \|\vec{a}\|\|\vec{b}\| \sin\theta$ $= (10)(2)\left(\frac{4}{5}\right) = 16$ $\therefore$ area of parallelogram $= 16$	½ ½ ½ ½
Q25.	Find the angle between the lines $\vec{r} = (3 + 2\lambda)\hat{i} - (2 - 2\lambda)\hat{j} + (6 + 2\lambda)\hat{k}$ and $\vec{r} = (2\hat{j} - 5\hat{k}) + \mu(6\hat{i} + 3\hat{j} + 2\hat{k})$.	

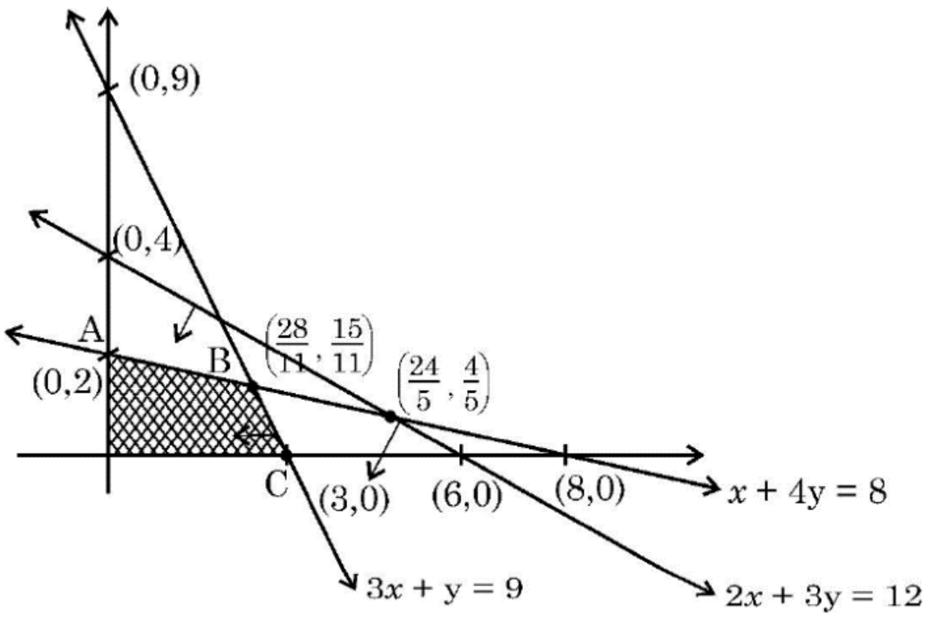
Ans	Given lines are: $\vec{r}=\left(3\hat{i}-2\hat{j}+6\hat{k}\right)+\lambda\left(2\hat{i}+2\hat{j}+2\hat{k}\right)$ and $\vec{r}=\left(2\hat{j}-5\hat{k}\right)+\mu\left(6\hat{i}+3\hat{j}+2\hat{k}\right)$ Let θ be the angle between these two lines. $\cos\theta=\frac{2(6)+2(3)+2(2)}{\sqrt{4+4+4}\sqrt{36+9+4}}=\frac{22}{2\sqrt{3}\times 7}$ $\Rightarrow \cos\theta=\frac{11}{21}\sqrt{3} \Rightarrow \theta=\cos^{-1}\left(\frac{11}{21}\sqrt{3}\right)$	½ 1 ½

SECTION C

This section comprises short answer (SA) type questions of **3 marks each**.

Q26.	Find the maximum slope of the curve $y=-x^3+3x^2+9x-30$.	
Ans	$y=-x^3+3x^2+9x-30$ Slope of the curve, $m=\frac{dy}{dx}=-3x^2+6x+9$ $\Rightarrow \frac{dm}{dx}=-6x+6$ For maximum/ minimum slope, put $\frac{dm}{dx}=0$ $\Rightarrow x=1$ As $\frac{d^2m}{dx^2}=-6<0 \therefore m$ is maximum at $x=1$ Maximum slope $=-3(1)^2+6(1)+9=12$	 1 ½ ½ ½ ½
Q27.	(a) Find : $\int\sqrt{4x^2-4x+10}\ dx$ **OR** (b) Evaluate : $\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}\,dx$	
Ans(a)	$I=\int\sqrt{4x^2-4x+10}\,dx$ $=\int\sqrt{(2x-1)^2+(3)^2}\,dx$ $=\frac{1}{2}\left[\left(\frac{2x-1}{2}\right)\sqrt{4x^2-4x+10}+\frac{9}{2}\log\left\|(2x-1)+\sqrt{4x^2-4x+10}\right\|\right]+C$	 1 1+1

OR

Ans(b)	$I=\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx \qquad ...(i)$ $=\int_0^{\pi}\frac{(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)}dx$ $I=\int_0^{\pi}\frac{(\pi-x)\sin x}{1+\cos^2 x}dx \qquad ...(ii)$ Adding (i) and (ii) $2I=\int_0^{\pi}\frac{\pi\sin x}{1+\cos^2 x}dx$ $\Rightarrow I=\frac{\pi}{2}\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}dx$ Put $\cos x=t\Rightarrow-\sin x\,dx=dt$ $I=-\frac{\pi}{2}\int_1^{-1}\frac{dt}{1+t^2}$ $=\pi\left[\tan^{-1}t\right]_0^1=\frac{\pi^2}{4}$	½ ½ ½ ½ 1
Q28.	Solve the following LPP graphically : Maximize Z = 2x + 3y subject to the constraints $x + 4y \le 8$ $2x + 3y \le 12$ $3x + y \le 9$ $x \ge 0, y \ge 0.$	
Ans		For correct graph and shading 2

	<table><tr><th>Corner Point</th><th>Value of $Z=2x+3y$</th></tr><tr><td>$O(0,0)$</td><td>0</td></tr><tr><td>$A(0,2)$</td><td>6</td></tr><tr><td>$B\left(\frac{28}{11},\frac{15}{11}\right)$</td><td>$\frac{101}{11}$ Maximum</td></tr><tr><td>$C(3,0)$</td><td>6</td></tr></table> $Z_{\max}=\frac{101}{11}$ when $x=\frac{28}{11}, y=\frac{15}{11}$	For correct table 1
Q29.	(a) Find the general solution of the differential equation $(2x^2 + y)\, dx = x\, dy$. **OR** (b) For the differential equation $\frac{dy}{dx} - \frac{y}{x} + \text{cosec}\left(\frac{y}{x}\right) = 0$, find the particular solution, given that $y = 0$ when $x = 1$.	
Ans(a)	$(2x^2+y)dx = x\,dy$ $\Rightarrow \frac{dy}{dx}-\frac{1}{x}.y=2x$ $\text{I.F.}=e^{-\int\frac{1}{x}dx}=\frac{1}{x}$ Solution is given by, $y.\left(\frac{1}{x}\right)=\int 2x.\frac{1}{x}dx$ $\Rightarrow \frac{y}{x}=2x+C \ or \ y=2x^2+Cx$	 1 1 ½ ½
	OR	
Ans(b)	$\frac{dy}{dx}=\frac{y}{x}-\cos ec\left(\frac{y}{x}\right)$ Put $\frac{y}{x}=v$ i.e. $y=vx$ $\Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$ The differential equation reduces to $v+x\frac{dv}{dx}=v-\cos ecv$	 ½ ½

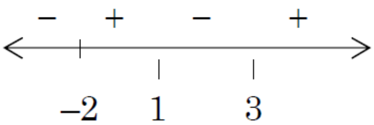
	$\sin v\, dv = -\frac{1}{x}dx \Rightarrow \int \sin v\, dv = -\int \frac{dx}{x}$ $\Rightarrow -\cos v = -\log\lvert x\rvert - C$ or, $\cos v = \log\lvert x\rvert + C'$ $\Rightarrow \cos\left(\frac{y}{x}\right) = \log\lvert x\rvert + C'$ $y = 0, x = 1$ gives $C' = 1$ $\therefore$ Solution is given by $\cos\left(\frac{y}{x}\right) = \log\lvert x\rvert + 1$	1 ½ ½
Q30.	If $\hat{a}$, $\hat{b}$ and $\hat{c}$ are unit vectors such that $\hat{a}.\hat{b} = \hat{a}.\hat{c} = 0$ and the angle between $\hat{b}$ and $\hat{c}$ is $\frac{\pi}{6}$, then prove that $\hat{a} = \pm 2(\hat{b} \times \hat{c})$.	
Ans)	$\hat{a}.\hat{b} = 0 \Rightarrow \hat{a} \perp \hat{b}, \hat{a}.\hat{c} = 0 \Rightarrow \hat{a} \perp \hat{c}$ $\Rightarrow \hat{a}$ is perpendicular to both $\hat{b}$ and $\hat{c} \Rightarrow \hat{a} \parallel (\hat{b} \times \hat{c})$ Let $\hat{a} = \lambda(\hat{b} \times \hat{c})$ $\Rightarrow \lvert\hat{a}\rvert = \lvert\lambda\rvert \lvert(\hat{b}\times\hat{c})\rvert \Rightarrow \lvert\hat{a}\rvert = \lvert\lambda\rvert \lvert\hat{b}\rvert \lvert\hat{c}\rvert \sin\frac{\pi}{6}$ $\Rightarrow \lvert\lambda\rvert = 2 \Rightarrow \lambda = \pm 2$ $\therefore \hat{a} = \pm 2(\hat{b} \times \hat{c})$	1 ½ 1 ½
Q31.	(a) Four students of class XII are given a problem to solve independently. Their chances of solving the problem respectively are $\frac{1}{2}, \frac{1}{3}, \frac{2}{3}$ and $\frac{1}{5}$. Find the probability that at most one of them will solve the problem. **OR** (b) The probability distribution of a random variable X is given below : X: 1, 2, 4, 2k, 3k, 5k P(X): $\frac{1}{2}, \frac{1}{5}, \frac{3}{25}, \frac{1}{10}, \frac{1}{25}, \frac{1}{25}$ Find k, if E(X) = 2·94 and also find P(X ≤ 4).	
Ans(a)	P(at most one of them will solve the problem) $= P$(none of them solves the problem) $+ P$(only one of them solves the problem) $= \left(\frac{1}{2}\times\frac{2}{3}\times\frac{1}{3}\times\frac{4}{5}\right) + \left(\frac{1}{2}\times\frac{2}{3}\times\frac{1}{3}\times\frac{4}{5} + \frac{1}{2}\times\frac{1}{3}\times\frac{1}{3}\times\frac{4}{5} + \frac{1}{2}\times\frac{2}{3}\times\frac{2}{3}\times\frac{4}{5} + \frac{1}{2}\times\frac{2}{3}\times\frac{1}{3}\times\frac{1}{5}\right)$ $= \frac{19}{45}$	2½ ½

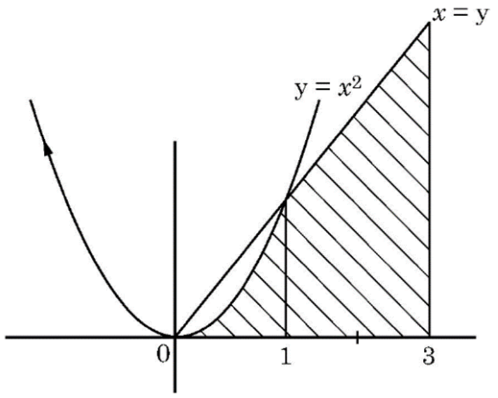
OR		
Ans(b)	$E(X) = 2.94$ $\Rightarrow 1\left(\frac{1}{2}\right) + 2\left(\frac{1}{5}\right) + 4\left(\frac{3}{25}\right) + 2k\left(\frac{1}{10}\right) + 3k\left(\frac{1}{25}\right) + 5k\left(\frac{1}{25}\right) = 2.94$ $\Rightarrow k = \frac{1.56}{0.52} \Rightarrow k = 3$ Now, $P(X \leq 4) = P(X = 1) + P(X = 2) + P(X = 4)$ $= \frac{1}{2} + \frac{1}{5} + \frac{3}{25}$ $= \frac{41}{50}$	1 1 ½ ½

SECTION D

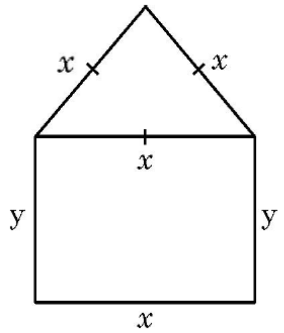
This section comprises long answer (LA) type questions of **5 marks each**.

Q32.	If $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$, find A^{-1}. Using A^{-1}, solve the given system of equations $3x + 4y + 7z = 14$; $2x - y + 3z = 4$; $x + 2y - 3z = 0$.	
Ans	$\lvert A \rvert = 3(-3) - 2(-26) + 1(19) = 62 \neq 0 \Rightarrow A^{-1}$ exists. cofactor Matrix $= \begin{bmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{bmatrix}$ $adjA = \begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{bmatrix}$ $A^{-1} = \frac{1}{62}\begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{bmatrix}$ Given system of equations can be written as $A'.X = B$ where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}$ Now, $A'.X = B \Rightarrow X = (A')^{-1}.B$ $\Rightarrow X = (A^{-1})'.B = \frac{1}{62}\begin{bmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{bmatrix}\begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}$ $= \frac{1}{62}\begin{bmatrix} 62 \\ 62 \\ 62 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\Rightarrow x = 1, y = 1, z = 1$	½ 2 ½ ½ ½ 1

Q33.	(a) If $y = \cos x^2 + \cos^2 x + \cos^2 (x^2) + \cos (x^x)$, find $\frac{dy}{dx}$. **OR** (b) Find the intervals in which the function given by $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$ is : (i) strictly increasing. (ii) strictly decreasing.	
Ans(a)	$y = \cos x^2 + \cos^2 x + \cos^2(x^2) + \cos(x^x)$ $\frac{d}{dx}(\cos x^2) = -2x \sin x^2$ $\frac{d}{dx}(\cos^2 x) = 2\cos x\,(-\sin x) = -2\sin x\cos x$ $\frac{d}{dx}(\cos^2(x^2)) = 2\cos(x^2)\,(-\sin(x^2))(2x) = -4x \sin x^2 \cos x^2$ $\frac{d}{dx}(\cos(x^x)) = -\sin(x^x)\,[x^x(1 + \log x)]$ $\frac{dy}{dx} = -2x\sin x^2 - 2\sin x\cos x - 4x\sin x^2\cos x^2 - \sin(x^x)\,[x^x(1 + \log x)]$	 1 1 1 1½ ½
	OR	
Ans(b)	$f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$ $\Rightarrow f'(x) = \frac{6}{5}x^3 - \frac{12}{5}x^2 - 6x + \frac{36}{5} = \frac{6}{5}(x^3 - 2x^2 - 5x + 6)$ $= \frac{6}{5}(x-1)(x+2)(x-3)$  For strictly inc/dec, put $f'(x) = 0$ $\Rightarrow x = 1, -2, 3$ (i) f is strictly increasing when $x \in (-2, 1) \cup (3, \infty)$ (ii) f is strictly decreasing when $x \in (-\infty, -2) \cup (1, 3)$ Note: Closed intervals are also acceptable.	 2 1 1 1
Q34.	Using integration, find the area of the region $\{(x, y) : 0 \le y \le x^2,\ 0 \le y \le x,\ 0 \le x \le 3\}$.	

Ans	 Required Area $=\int_0^1 x^2\,dx+\int_1^3 x\,dx$ $=\left.\frac{x^3}{3}\right]_0^1+\left.\frac{x^2}{2}\right]_1^3$ $=\frac{1}{3}+4=\frac{13}{3}$	1 mark for correct figure 1+1 1 1
Q35.	(a) Find the shortest distance between the lines l_1 and l_2 given by : $l_1 : \vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(4\hat{i} + 6\hat{j} + 12\hat{k})$ and $l_2 : \vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(6\hat{i} + 9\hat{j} + 18\hat{k})$ **OR** (b) Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1}$ intersect. Also, find their point of intersection.	
Ans(a)	Given lines are : $\vec{r}=\left(\hat{i}+2\hat{j}-4\hat{k}\right)+2\lambda\left(2\hat{i}+3\hat{j}+6\hat{k}\right)$ and $\vec{r}=\left(\hat{i}+2\hat{j}-4\hat{k}\right)+3\mu\left(2\hat{i}+3\hat{j}+6\hat{k}\right)$ Clearly, the given lines are parallel. Here, $\vec{a}_1=\hat{i}+2\hat{j}-4\hat{k}$, $\vec{a}_2=\hat{i}+2\hat{j}-4\hat{k}$ and $\vec{b}=2\hat{i}+3\hat{j}+6\hat{k}$ $\vec{a}_2-\vec{a}_1=2\hat{i}+\hat{j}-\hat{k}$ $\left(\vec{a}_2-\vec{a}_1\right)\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & -1\\ 2 & 3 & 6\end{vmatrix}=9\hat{i}-14\hat{j}+4\hat{k}$ $\therefore \left\|\left(\vec{a}_2-\vec{a}_1\right)\times\vec{b}\right\|=\sqrt{81+196+16}=\sqrt{293}$ Also, $\left\|\vec{b}\right\|=\sqrt{4+9+36}=7$ S.D. $=\frac{\left\|\left(\vec{a}_2-\vec{a}_1\right)\times\vec{b}\right\|}{\left\|\vec{b}\right\|}$ $=\frac{\sqrt{293}}{7}$	 1 ½ 1 ½ 1 ½ ½

OR		
Ans(b)	Let the given lines be $l_1: \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$ and $l_2: \frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1} = \mu$ Any point on the line l_1 is $(2\lambda+1, 3\lambda+2, 4\lambda+3)$ Any point on the line l_2 is $(5\mu+4, 2\mu+1, \mu)$ For the given lines to intersect, there must be a common point. $\therefore 2\lambda+1 = 5\mu+4 \Rightarrow 2\lambda-5\mu=3 \quad ...(i)$ $3\lambda+2 = 2\mu+1 \Rightarrow 3\lambda-2\mu=-1 \quad ...(ii)$ $4\lambda+3 = \mu \Rightarrow 4\lambda-\mu=-3 \quad ...(iii)$ Solving (i) and (ii) gives, $\lambda = \mu = -1$ We notice that $\lambda = \mu = -1$ also satisfies equation (iii) $\therefore$ The given lines intersect. Point of intersection is $(2(-1)+1, 3(-1)+2, 4(-1)+3)$ i.e. $(-1,-1,-1)$	 1 1 1 1 1
SECTION E This section comprises 3 case study-based questions of **4 marks each**.		
Q36.	**Case Study – 1** A window is in the form of a rectangle surmounted by an equilateral triangle on its length. Let the rectangular part have length and breadth x and y metres respectively. Based on the given information, answer the following questions : (i) If the perimeter of the window is 12 m, find the relation between x and y. *1* (ii) Using the expression obtained in (i), write an expression for the area of the window as a function of x only. *1* (iii) (a) Find the dimensions of the rectangle that will allow maximum light through the window. (use expression obtained in (ii)) *2* **OR** (iii) (b) If it is given that the area of the window is 50 m^2, find an expression for its perimeter in terms of x. *2*	

Ans	(i) Perimeter $(P) = 3x + 2y = 12$ 	1
	(ii) Area $(A) = xy + \frac{\sqrt{3}}{4}x^2$ $= x\left(\frac{12-3x}{2}\right) + \frac{\sqrt{3}}{4}x^2$ $= 6x - \frac{3}{2}x^2 + \frac{\sqrt{3}}{4}x^2$	1
	$(iii)(a)\ \frac{dA}{dx} = 6 - 3x + \frac{\sqrt{3}}{2}x$	½
	For maximum light, $\frac{dA}{dx} = 0$ $\Rightarrow 6 - 3x + \frac{\sqrt{3}}{2}x = 0 \Rightarrow x = \frac{12}{6-\sqrt{3}}\,m$	½
	Also, $\frac{d^2A}{dx^2} = -3 + \frac{\sqrt{3}}{2} < 0 \therefore A$ is maximum when $x = \frac{12}{6-\sqrt{3}}\,m$	½
	Now, $y = \frac{12-3x}{2} = 6 - \frac{3}{2}\left(\frac{12}{6-\sqrt{3}}\right) = \frac{18-6\sqrt{3}}{6-\sqrt{3}}\,m$	½
	OR	
	$(iii)(b)\ xy + \frac{\sqrt{3}}{4}x^2 = 50$	½
	$\Rightarrow y = \frac{50}{x} - \frac{\sqrt{3}}{4}x$	1
	Now, $P = 3x + 2y$ $= 3x + 2\left(\frac{50}{x} - \frac{\sqrt{3}}{4}x\right)m$	½

Q37.	**Case Study – 2** During the festival season, there was a mela organized by the Resident Welfare Association at a park, near the society. The main attraction of the mela was a huge swing installed at one corner of the park. The swing is traced to follow the path of a parabola given by $x^2 = y$. Based on the above information, answer the following questions : (i) Let $f : N \longrightarrow R$ is defined by $f(x) = x^2$. What will be the range ? *1* (ii) Let $f : N \longrightarrow N$ is defined by $f(x) = x^2$. Check if the function is injective or not. *1* (iii) (a) Let $f : \{1, 2, 3, 4 .....\} \longrightarrow \{1, 4, 9, 16 .....\}$ be defined by $f(x) = x^2$. Prove that the function is bijective. *2* **OR** (iii) (b) Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ is defined by $f(x) = x^2$. Show that f is neither injective nor surjective. *2*	
Ans	$(i)\ R_f = \{1,4,9,16,...\}$ i.e. set of perfect squares of natural numbers. (ii) Let $x_1, x_2 \in N$ (domain) Let $f(x_1) = f(x_2)$ $\Rightarrow x_1^2 = x_2^2$ $\Rightarrow x_1 = \pm x_2$ $\Rightarrow x_1 = x_2$ as $x_1, x_2 \in N$ $\therefore f$ is injective. $(iii)(a)\ f(x) = x^2$ Let $x_1, x_2 \in \{1,2,3,4,...\}$ Let $f(x_1) = f(x_2)$ $\Rightarrow x_1^2 = x_2^2$ $\Rightarrow x_1 = x_2$ $\therefore f$ is one-one. As Co-domain = Range = $\{1,4,9,16,...\}$ $\therefore f$ is onto. Since, f is one-one and onto, so f is bijective. OR $(iii)(b)\ f : R \to R,\ f(x) = x^2$ $-1, 1 \in R$ (domain) As $f(-1) = f(1) = 1$ but $-1 \neq 1$ $\therefore f$ is not injective. Co-domain $= R$, but Range $= [0, \infty)$ Since Co-domain $\neq$ Range, f is not surjective.	1 1 1 1 1 1

Q38.	**Case Study – 3** Two persons are competing for a position on the Managing Committee of an organisation. The probabilities that the first and the second person will be appointed are 0·5 and 0·6 respectively. Also, if the first person gets appointed, then the probability of introducing waste treatment plant is 0·7 and the corresponding probability is 0·4, if the second person gets appointed. Based on the above information, answer the following questions : (i) What is the probability that the waste treatment plant is introduced ? *2* (ii) After the selection, if the waste treatment plant is introduced, what is the probability that the first person had introduced it ? *2*	
Ans	E_1 :Event that the first person is appointed. E_2 :Event that the second person is appointed. A: Event that the waste treatment plant is introducted. Here, $P(E_1)=0.5, P(E_2)=0.6$ $P(A\|E_1)=0.7, P(A\|E_2)=0.4$ $(i)\ P(\text{waste treatment plant is introducted})$ $=P(E_1)P(A\|E_1)+P(E_2)P(A\|E_2)$ $=(0.5)(0.7)+(0.6)(0.4)$ $=0.35+0.24=0.59$ $(ii)\ P(E_1\|A)=\dfrac{P(E_1)P(A\|E_1)}{P(E_1)P(A\|E_1)+P(E_2)P(A\|E_2)}$ $=\dfrac{(0.5)(0.7)}{(0.5)(0.7)+(0.6)(0.4)}$ $=\dfrac{0.35}{0.59}=\dfrac{35}{59}$ Note: Full marks to be awarded, in case a student writes "Sum of probabilities of selecting first person and second person should not be greater than 1".	1 1 1 ½ ½