MARKING SCHEME

SENIOR SECONDARY SCHOOL EXAMINATION 2024-25

MATHEMATICS (Code–041)

[Paper Code: 65/4/1]

Q. No.	EXPECTED ANSWER / VALUE POINTS	Marks
	SECTION - A Questions no. 1 to 18 are multiple choice questions (MCQs) of **1 mark each**.	
Q1.	The principal value of $\sin^{-1}\left(\sin\left(-\frac{10\pi}{3}\right)\right)$ is : (A) $-\frac{2\pi}{3}$ (B) $-\frac{\pi}{3}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$	
A1.	(C) $\frac{\pi}{3}$	1
Q2.	If A and B are square matrices of same order such that AB = A and BA = B, then $A^2 + B^2$ is equal to : (A) A + B (B) BA (C) 2 (A + B) (D) 2BA	
A2.	(A) A + B	1
Q3.	For real x, let $f(x) = x^3 + 5x + 1$. Then : (A) f is one-one but not onto on R (B) f is onto on R but not one-one (C) f is one-one and onto on R (D) f is neither one-one nor onto on R	
A3.	(C) f is one-one and onto on R	1
Q4.	If $y = \sin^{-1} x$, then $(1 - x^2)\frac{d^2y}{dx^2}$ is equal to : (A) $x\frac{dy}{dx}$ (B) $-x\frac{dy}{dx}$ (C) $x^2\frac{dy}{dx}$ (D) $-x^2\frac{dy}{dx}$	
A4.	(A) $x\frac{dy}{dx}$	1

Q5.	The values of λ so that $f(x) = \sin x - \cos x - \lambda x + C$ decreases for all real values of x are : (A) $1 < \lambda < \sqrt{2}$ (B) $\lambda \geq 1$ (C) $\lambda \geq \sqrt{2}$ (D) $\lambda < 1$	
A5.	(C) $\lambda \geq \sqrt{2}$	1
Q6.	If P is a point on the line segment joining (3, 6, –1) and (6, 2, – 2) and y-coordinate of P is 4, then its z-coordinate is : (A) $-\frac{3}{2}$ (B) 0 (C) 1 (D) $\frac{3}{2}$	
A6.	(A) $-\frac{3}{2}$	1
Q7.	If M and N are square matrices of order 3 such that det (M) = m and MN = mI, then det (N) is equal to : (A) – 1 (B) 1 (C) $-m^2$ (D) m^2	
A7.	(D) m^2	1
Q8.	If $f(x) = \begin{cases} 3x - 2, & 0 < x \leq 1 \\ 2x^2 + ax, & 1 < x < 2 \end{cases}$ is continuous for $x \in (0, 2)$, then a is equal to : (A) – 4 (B) $-\frac{7}{2}$ (C) – 2 (D) – 1	
A8.	(D) – 1	1

<table>
<tr><td>Q9.</td><td colspan="2">If $f : N \to W$ is defined as
$$f(n) = \begin{cases} \frac{n}{2}, & \text{if n is even} \\ 0, & \text{if n is odd} \end{cases},$$
then f is :
(A) injective only (B) surjective only
(C) a bijection (D) neither surjective nor injective</td></tr>
<tr><td>A9.</td><td>(B) surjective only</td><td>1</td></tr>
<tr><td>Q10.</td><td colspan="2">The matrix $\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & -7 \\ 2 & 7 & 0 \end{bmatrix}$ is a :
(A) diagonal matrix (B) symmetric matrix
(C) skew symmetric matrix (D) scalar matrix</td></tr>
<tr><td>A10.</td><td>(C) skew symmetric matrix</td><td>1</td></tr>
<tr><td>Q11.</td><td colspan="2">If the sides AB and AC of Δ ABC are represented by vectors $\hat{j} + \hat{k}$ and $3\hat{i} - \hat{j} + 4\hat{k}$ respectively, then the length of the median through A on BC is :
(A) $2\sqrt{2}$ units (B) $\sqrt{18}$ units
(C) $\frac{\sqrt{34}}{2}$ units (D) $\frac{\sqrt{48}}{2}$ units</td></tr>
<tr><td>Q11.</td><td>(C) $\frac{\sqrt{34}}{2}$ units</td><td>1</td></tr>
<tr><td>Q12.</td><td colspan="2">The function f defined by
$$f(x) = \begin{cases} x, & \text{if } x \le 1 \\ 5, & \text{if } x > 1 \end{cases}$$
is not continuous at :
(A) $x = 0$ (B) $x = 1$
(C) $x = 2$ (D) $x = 5$</td></tr>
<tr><td>A12.</td><td>(B) $x = 1$</td><td>1</td></tr>
</table>

Q13.	If $f(x) = 2x + \cos x$, then $f(x)$: (A) has a maxima at $x = \pi$ (B) has a minima at $x = \pi$ (C) is an increasing function (D) is a decreasing function	
A13.	(C) is an increasing function	1
Q14.	$\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx$ is equal to : (A) $2(\sin x + x\cos\alpha) + C$ (B) $2(\sin x - x\cos\alpha) + C$ (C) $2(\sin x + 2x\cos\alpha) + C$ (D) $2(\sin x + \sin\alpha) + C$	
A14.	(A) $2(\sin x + x\cos\alpha) + C$	1
Q15.	The value of $\int_0^1 \frac{dx}{e^x + e^{-x}}$ is : (A) $-\frac{\pi}{4}$ (B) $\frac{\pi}{4}$ (C) $\tan^{-1} e - \frac{\pi}{4}$ (D) $\tan^{-1} e$	
A15.	(C) $\tan^{-1} e - \frac{\pi}{4}$	1
Q16.	The order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x \sin\left(\frac{dy}{dx}\right)$ are : (A) order 2, degree 2 (B) order 2, degree 1 (C) order 2, degree not defined (D) order 1, degree not defined	
A16.	(C) order 2, degree not defined	1
Q17.	The area of the region enclosed by the curve $y = \sqrt{x}$ and the lines $x = 0$ and $x = 4$ and x-axis is : (A) $\frac{16}{9}$ sq. units (B) $\frac{32}{9}$ sq. units (C) $\frac{16}{3}$ sq. units (D) $\frac{32}{3}$ sq. units	
A17.	(C) $\frac{16}{3}$ sq. units	1

Q18.	The corner points of the feasible region of a Linear Programming Problem are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). If Z = ax + by; (a, b > 0) be the objective function, and maximum value of Z is obtained at (0, 2) and (3, 0), then the relation between a and b is : (A) a = b (B) a = 3b (C) b = 6a (D) 3a = 2b	
A18.	(D) 3a = 2b	1

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Q19.	*Assertion (A) :* If A and B are two events such that $P(A \cap B) = 0$, then A and B are independent events. *Reason (R) :* Two events are independent if the occurrence of one does not effect the occurrence of the other.	
A19.	(D) Assertion (A) is false, but Reason (R) is true.	1
Q20.	*Assertion (A) :* In a Linear Programming Problem, if the feasible region is empty, then the Linear Programming Problem has no solution. *Reason (R) :* A feasible region is defined as the region that satisfies all the constraints.	
A20.	(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).	1

SECTION B

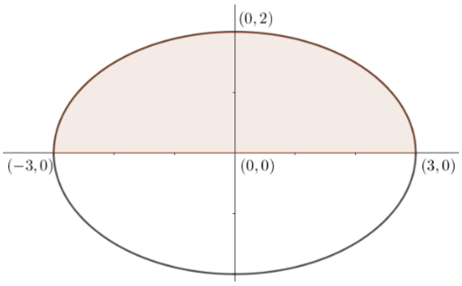
This section comprises very short answer (VSA) type questions of **2 marks each**.

Q21.	Let A and B be two square matrices of order 3 such that det (A) = 3 and det (B) = – 4. Find the value of det (– 6AB).	
A21.	$\lvert -6AB \rvert = (-6)^3 \lvert A \rvert \lvert B \rvert$ $= -216 \times 3 \times (-4) = 2592$	1 1

Q22.	(a) Find the least value of 'a' so that $f(x) = 2x^2 - ax + 3$ is an increasing function on $[2, 4]$. **OR** (b) If $f(x) = x + \frac{1}{x}, x \geq 1$, show that f is an increasing function.	
A22.(a)	$f(x) = 2x^2 - ax + 3 \Rightarrow f'(x) = 4x - a$ Now $2 \leq x \leq 4 \Rightarrow 8 - a \leq 4x - a \leq 16 - a$ For f to be an increasing function, $f'(x) \geq 0$ $\Rightarrow 8 - a \geq 0 \Rightarrow a \leq 8$ $\therefore$ Least value of a does not exist.	½ 1 ½
	OR	
A22.(b)	$f(x) = x + \frac{1}{x} \Rightarrow f'(x) = 1 - \frac{1}{x^2} = \frac{x^2 - 1}{x^2}$ Now $\frac{x^2 - 1}{x^2} \geq 0$ for all $x \geq 1$ $\Rightarrow f'(x) \geq 0 \Rightarrow f$ is an increasing function.	1 ½ ½
Q23.	(a) Simplify $\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$. **OR** (b) Find domain of $\sin^{-1}\sqrt{x-1}$.	
A23.(a)	Put $x = \tan\theta \Rightarrow \theta = \tan^{-1} x$ Now $\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$ $= \sin^{-1}\left(\frac{\tan\theta}{\sec\theta}\right) = \sin^{-1}(\sin\theta)$ $= \theta = \tan^{-1} x$	½ 1 ½
	OR	
A23.(b)	Here $-1 \leq \sqrt{x-1} \leq 1$ $\Rightarrow 0 \leq x - 1 \leq 1 \Rightarrow 1 \leq x \leq 2$ Hence, domain is $x \in [1, 2]$	1 1

Q24. Calculate the area of the region bounded by the curve $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the x-axis using integration.

A24.



(½ for correct figure)

$A = 2\times\frac{2}{3}\int_0^3\sqrt{9-x^2}\,dx$ ½

$=\frac{4}{3}\left[\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right)\right]_0^3$ ½

$=\frac{4}{3}\left[\left(0+\frac{9}{2}\sin^{-1}1\right)-0\right]$

$=3\pi$ ½

Q25. For the curve $y = 5x - 2x^3$, if x increases at the rate of 2 units/s, then how fast is the slope of the curve changing when $x = 2$?

A25.

$y=5x-2x^3$

Given $\frac{dx}{dt}=2$ units/s ½

slope of the curve $=\frac{dy}{dx}=5-6x^2=m$ ½

$\frac{dm}{dt}=-12x\frac{dx}{dt}=-12x(2)=-24x$ ½

at $x=2, \frac{dm}{dt}=-24(2)=-48$ ½

Hence, slope of curve is decreasing at the rate of 48

SECTION C

This section comprises short answer (SA) type questions of **3 marks each**.

Q26.

(a) If $f : R^+ \to R$ is defined as $f(x) = \log_a x$ $(a > 0$ and $a \neq 1)$, prove that f is a bijection.

(R^+ is a set of all positive real numbers.)

OR

(b) Let A = {1, 2, 3} and B = {4, 5, 6}. A relation R from A to B is defined as $R = \{(x, y) : x + y = 6, x \in A, y \in B\}$.

(i) Write all elements of R.

(ii) Is R a function ? Justify.

(iii) Determine domain and range of R.

A26.(a)	$f(x)=\log_a x \quad (a>0, a\neq 1)$ Let $x_1, x_2 \in R^+$ such that $f(x_1)=f(x_2)$ $\Rightarrow \log_a x_1 = \log_a x_2$ $\Rightarrow x_1 = x_2 \Rightarrow f$ is one-one. Let $f(x)=y \Rightarrow \log_a x = y \Rightarrow a^y = x$ $\therefore$ for every $y\in R$, there exists $x \in R^+$ $\therefore f$ is onto. f is a bijection.	1½ 1½
OR		
A26.(b)	$(i)\ R=\{(1,5),(2,4)\}$ $(ii)\ R$ is not a function as $3\in A$ do not have an image in co-domain. (iii) Domain of $R=\{1,2\}$, Range of $R=\{4,5\}$	1 1 1
Q27.	(a) Find k so that $f(x) = \begin{cases} \dfrac{x^2-2x-3}{x+1}, & x \neq -1 \\ k, & x=-1 \end{cases}$ is continuous at x = – 1. **OR** (b) Check the differentiability of function f(x) = x | x | at x = 0.	
A27(a).	$\lim_{x\to -1}\dfrac{x^2-2x-3}{x+1}=\lim_{x\to -1}\dfrac{(x-3)(x+1)}{x+1}=\lim_{x\to -1}(x-3)=-4$ Also, $f(-1)=k$ as f is continuous, $k=-4$	2 ½ ½
OR		
A27(b).	$f(x)=x\|x\|=\begin{cases}-x^2 & ,x\leq 0\\ x^2 & ,x>0\end{cases}$ $\text{LHD}=\lim_{h\to 0}\dfrac{f(0-h)-f(0)}{-h}=\lim_{h\to 0}\dfrac{-h^2-0}{-h}=0$ $\text{RHD}=\lim_{h\to 0}\dfrac{f(0+h)-f(0)}{h}=\lim_{h\to 0}\dfrac{h^2-0}{h}=0$ Since LHD = RHD, f is differentiable at $x=0$	1 1 ½ ½

Q28.	Evaluate : $$\int_{\pi/2}^{\pi} e^{x}\left(\frac{1-\sin x}{1-\cos x}\right)dx$$	
A28.	$I=\int_{\pi/2}^{\pi} e^{x}\left(\frac{1-\sin x}{1-\cos x}\right)dx$	
	$=\int_{\pi/2}^{\pi} e^{x}\left(\frac{1-2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^{2}\frac{x}{2}}\right)dx$	1
	$=\int_{\pi/2}^{\pi} e^{x}\left(\frac{1}{2}\operatorname{cosec}^{2}\frac{x}{2}-\cot\frac{x}{2}\right)dx$	
	$\therefore I=-\left[e^{x}\cot\frac{x}{2}\right]_{\pi/2}^{\pi}$	1
	$=-\left[e^{\pi}\cot\frac{\pi}{2}-e^{\frac{\pi}{2}}\cot\frac{\pi}{4}\right]$	
	$=e^{\frac{\pi}{2}}$	1
Q29.	(a) Find the probability distribution of the number of boys in families having three children, assuming equal probability for a boy and a girl. **OR** (b) A coin is tossed twice. Let X be a random variable defined as number of heads minus number of tails. Obtain the probability distribution of X and also find its mean.	
A29.(a)	Let X denote the random variable which counts the number of boys.	
	$X=0,1,2,3$	½
	$P(\text{Boy})=P(\text{Girl})=\frac{1}{2}$	½
	Required Probability Distribution	
	(table below)	2

X	0	1	2	3
$P(X)$	$\left(\frac{1}{2}\right)^{3}=\frac{1}{8}$	$3\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{2}=\frac{3}{8}$	$3\left(\frac{1}{2}\right)^{2}\left(\frac{1}{2}\right)=\frac{3}{8}$	$\left(\frac{1}{2}\right)^{3}=\frac{1}{8}$

OR

A29.(b)

Possible values of X are $-2, 0, 2$ ½

X	-2	0	2
$P(X)$	$\frac{1}{4}$	$\frac{2}{4}=\frac{1}{2}$	$\frac{1}{4}$

1½

$\text{Mean} = \sum XP(X) = -2\left(\frac{1}{4}\right) + 0\left(\frac{1}{2}\right) + 2\left(\frac{1}{4}\right) = 0$ 1

Q30.

Find the distance of the point (–1, –5, –10) from the point of intersection of the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$.

A30.

$l_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$

Any point on l_1 is $(2\lambda+1, 3\lambda+2, 4\lambda+3)$

$l_2 : \frac{x-4}{5} = \frac{y-1}{2} = \frac{z-0}{1} = \mu$

Any point on l_2 is $(5\mu+4, 2\mu+1, \mu)$ 1

For point of intersection,

$2\lambda+1 = 5\mu+4, 3\lambda+2 = 2\mu+1$

Solving, $\lambda = \mu = -1$ 1

Since, $\lambda = \mu = -1$ satisfy $4\lambda + 3 = \mu$

$\therefore$ Point of intersection is $(-1, -1, -1)$ ½

Now distance of $(-1, -5, -10)$ from $(-1, -1, -1)$ is:

$\sqrt{(-1+1)^2 + (-1+5)^2 + (-1+10)^2} = \sqrt{97}$ units ½

Q31.

Solve the following Linear Programming Problem using graphical method :

Maximise Z = 100x + 50y

subject to the constraints

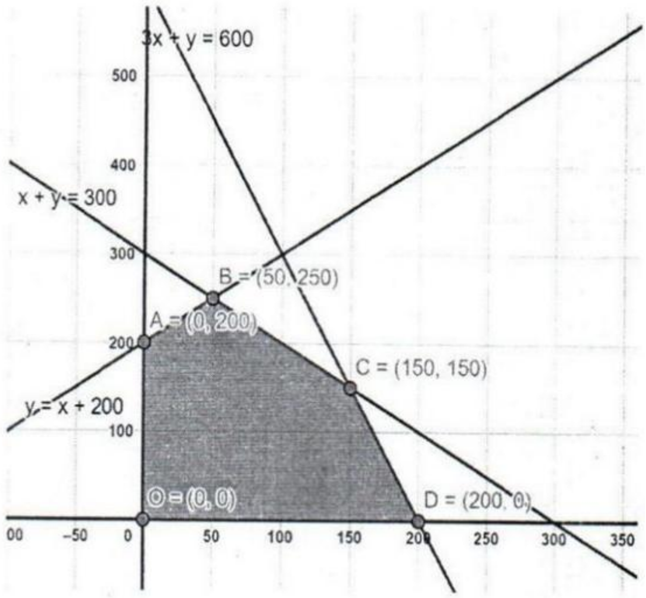
$3x + y \leq 600$

$x + y \leq 300$

$y \leq x + 200$

$x \geq 0, y \geq 0$

A31.



For correct graph and shading $1\frac{1}{2}$

Corner Point	Value of $Z = 100x + 50y$
$O(0,0)$	0
$A(0,200)$	10000
$B(50,250)$	17500
$C(150,150)$	22500
$D(200,0)$	20000

For correct table 1

$Z_{max} = 22500$ when $x = 150,\ y = 150$ — $\frac{1}{2}$

SECTION D

This section comprises long answer (LA) type questions of **5 marks each**.

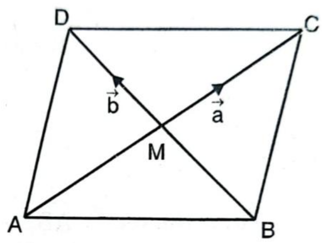
Q32. If A is a 3 × 3 invertible matrix, show that for any scalar k ≠ 0, $(kA)^{-1} = \frac{1}{k}A^{-1}$. Hence calculate $(3A)^{-1}$, where

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$$

A32.	Consider $(kA)\left(\frac{1}{k}A^{-1}\right)=k.\frac{1}{k}\left(A.A^{-1}\right)=I$ $\Rightarrow kA$ and $\frac{1}{k}A^{-1}$ are inverse of each other. $\therefore (kA)^{-1}=\frac{1}{k}A^{-1}$	1
	$\therefore (3A)^{-1}=\frac{1}{3}A^{-1}$	
	Here, $\lvert A\rvert = 4\neq 0 \therefore A^{-1}$ exists.	1
	$adjA=\begin{bmatrix}3 & 1 & -1\\ 1 & 3 & 1\\ -1 & 1 & 3\end{bmatrix}$	2
	$\therefore A^{-1}=\frac{1}{\lvert A\rvert}.adjA=\frac{1}{4}\begin{bmatrix}3 & 1 & -1\\ 1 & 3 & 1\\ -1 & 1 & 3\end{bmatrix}$	½
	$\therefore (3A)^{-1}=\frac{1}{12}\begin{bmatrix}3 & 1 & -1\\ 1 & 3 & 1\\ -1 & 1 & 3\end{bmatrix}$	½
Q33.	The relation between the height of the plant (y cm) with respect to exposure to sunlight is governed by the equation $y = 4x - \frac{1}{2}x^2$, where x is the number of days exposed to sunlight. (i) Find the rate of growth of the plant with respect to sunlight. *2* (ii) In how many days will the plant attain its maximum height ? What is the maximum height ? *3*	
A33.	$(i)\ y=4x-\frac{1}{2}x^2 \Rightarrow \frac{dy}{dx}=(4-x)$ cm/day	2
	(ii) For maximum height, $\frac{dy}{dx}=0 \Rightarrow x=4$ days	1
	as $\frac{d^2y}{dx^2}<0$, number of days $=4$	1
	Now, Maximum height $=y(4)=16-\frac{1}{2}(16)=8$ cm	1

Q34.	(a) Find : $$\int \frac{\cos x}{(4+\sin^2 x)(5-4\cos^2 x)}\, dx$$ **OR** (b) Evaluate : $$\int_0^{\pi} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$$	
A34.(a)	$I=\int \frac{\cos x}{\left(4+\sin^2 x\right)\left(5-4\cos^2 x\right)}dx$	
	$=\int \frac{\cos x}{\left(4+\sin^2 x\right)\left(1+4\sin^2 x\right)}dx$	1
	$\sin x = t$ gives	½
	$I=\int \frac{dt}{\left(4+t^2\right)\left(1+4t^2\right)}$	
	$=-\frac{1}{15}\int \frac{dt}{4+t^2}+\frac{4}{15}\int \frac{dt}{1+4t^2}$ $(\because \text{using Partial Fraction})$	2
	$=-\frac{1}{30}\tan^{-1}\left(\frac{t}{2}\right)+\frac{2}{15}\tan^{-1}\left(2t\right)+C$	1
	$=-\frac{1}{30}\tan^{-1}\left(\frac{\sin x}{2}\right)+\frac{2}{15}\tan^{-1}\left(2\sin x\right)+C$	½

OR

A34.(b)	$I=\int_0^{\pi}\frac{dx}{a^2\cos^2 x+b^2\sin^2 x}$ $=2\int_0^{\pi/2}\frac{dx}{a^2\cos^2 x+b^2\sin^2 x}$ $=2\int_0^{\pi/2}\frac{\sec^2 x}{a^2+b^2\tan^2 x}dx$ $\tan x=t$ gives $I=2\int_0^{\infty}\frac{dt}{a^2+b^2t^2}$ $=\frac{2}{b^2}.\frac{b}{a}\tan^{-1}\left(\frac{bt}{a}\right)\Big]_0^{\infty}$ $=\frac{\pi}{ab}$	 ½ 1 ½ ½ 1½ 1
Q35 .	(a) Show that the area of a parallelogram whose diagonals are represented by $\vec{a}$ and $\vec{b}$ is given by $\frac{1}{2}\mid\vec{a}\times\vec{b}\mid$. Also find the area of a parallelogram whose diagonals are $2\hat{i}-\hat{j}+\hat{k}$ and $\hat{i}+3\hat{j}-\hat{k}$. **OR** (b) Find the equation of a line in vector and cartesian form which passes through the point (1, 2, – 4) and is perpendicular to the lines $\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7}$, and $\vec{r}=15\hat{i}+29\hat{j}+5\hat{k}+\mu(3\hat{i}+8\hat{j}-5\hat{k})$.	
A35.(a)	Let $ABCD$ be the parallelogram with diagonals $\overrightarrow{AB}=\vec{a}$ and $\overrightarrow{BD}=\vec{b}$. $\therefore\overrightarrow{AB}=\frac{1}{2}(\vec{a}-\vec{b})$ and $\overrightarrow{AD}=\frac{1}{2}(\vec{a}+\vec{b})$  Area of $ABCD$ $=\left\|\overrightarrow{AB}\times\overrightarrow{AD}\right\|$ $=\left\|\frac{1}{2}(\vec{a}-\vec{b})\times\frac{1}{2}(\vec{a}+\vec{b})\right\|$ $=\frac{1}{4}\left\|\vec{a}\times\vec{a}+\vec{a}\times\vec{b}-\vec{b}\times\vec{a}-\vec{b}\times\vec{b}\right\|$	½ ½ ½

	$=\frac{1}{4}\left\|\vec{a}\times\vec{b}+\vec{a}\times\vec{b}\right\| \quad (\because \vec{a}\times\vec{a}=\vec{0})$ $=\frac{1}{4}\left\|2\left(\vec{a}\times\vec{b}\right)\right\|$ $=\frac{1}{2}\left\|\vec{a}\times\vec{b}\right\|$ Given $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+3\hat{j}-\hat{k}$ Area of parallelogram $=\frac{1}{2}\left\|\vec{a}\times\vec{b}\right\|$ Now $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&-1&1\\1&3&-1\end{vmatrix}=-2\hat{i}+3\hat{j}+7\hat{k}$ $\left\|\vec{a}\times\vec{b}\right\|=\sqrt{62}$ Area of parallelogram $=\frac{1}{2}\sqrt{62}$	½ 2 ½ ½
OR		
A35.(b)	Given lines are $\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7}$ and $\vec{r}=\left(15\hat{i}+29\hat{j}+5\hat{k}\right)+\mu\left(3\hat{i}+8\hat{j}-5\hat{k}\right)$ The first line in vector form is $\vec{r}=\left(8\hat{i}-19\hat{j}+10\hat{k}\right)+\lambda\left(3\hat{i}-16\hat{j}+7\hat{k}\right)$ $\vec{a}_1=8\hat{i}-19\hat{j}+10\hat{k}, \vec{a}_2=15\hat{i}+29\hat{j}+5\hat{k}$ $\vec{b}_1=3\hat{i}-16\hat{j}+7\hat{k}$, $\vec{b}_2=3\hat{i}+8\hat{j}-5\hat{k}$ $\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\3&-16&7\\3&8&-5\end{vmatrix}=24\hat{i}+36\hat{j}+72\hat{k}$ $\therefore$ Equation of line passing through $(1,2,-4)$ and parallel to $\vec{b}$ is $\vec{r}=\left(\hat{i}+2\hat{j}-4\hat{k}\right)+t\left(24\hat{i}+36\hat{j}+72\hat{k}\right)$ or $\vec{r}=\left(\hat{i}+2\hat{j}-4\hat{k}\right)+t'\left(2\hat{i}+3\hat{j}+6\hat{k}\right)$ Cartesian form of line is $\frac{x-1}{24}=\frac{y-2}{36}=\frac{z+4}{72}$ or $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+4}{6}$	1 1 1 1 1

SECTION E

This section comprises 3 case study-based questions of **4 marks each**.

Q36. Some students are having a misconception while comparing decimals. For example, a student may mention that $78{\cdot}56 > 78{\cdot}9$ as $7856 > 789$. In order to assess this concept, a decimal comparison test was administered to the students of class VI through the following question : In the recently held Sports Day in the school, 5 students participated in a javelin throw competition. The distances to which they have thrown the javelin are shown below in the table :

Name of student	Distance of javelin (in meters)
Ajay	47·7
Bijoy	47·07
Kartik	43·09
Dinesh	43·9
Devesh	45·2

The students were asked to identify who has thrown the javelin the farthest.

Based on the test attempted by the students, the teacher concludes that 40% of the students have the misconception in the concept of decimal comparison and the rest do not have the misconception. 80% of the students having misconception answered Bijoy as the correct answer in the paper. 90% of the students who are identified with not having misconception, did not answer Bijoy as their answer.

On the basis of the above information, answer the following questions :

(i) What is the probability of a student not having misconception but still answers Bijoy in the test ? *1*

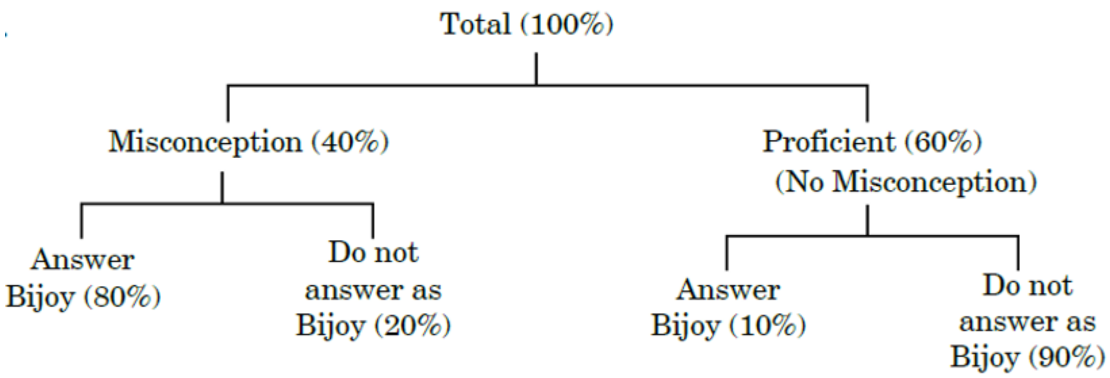
(ii) What is the probability that a randomly selected student answers Bijoy as his answer in the test ? *1*

(iii) (a) What is the probability that a student who answered as Bijoy is having misconception ? *2*

OR

(iii) (b) What is the probability that a student who answered as Bijoy is amongst students who do not have the misconception ? *2*

A36.



	Let E_1 : Student has a misconception E_2 :Student does not have misconception A: Student answered Bijoy as correct $\therefore P(E_1)=\frac{40}{100}, \text{P}(E_2)=\frac{60}{100}$ $P(A\|E_1)=\frac{80}{100}, P(A\|E_2)=\frac{10}{100}$ $P(\bar{A}\|E_1)=\frac{20}{100}, P(\bar{A}\|E_2)=\frac{90}{100}$ $(i) P(A\|E_2)=\frac{10}{100} \text{ or } \frac{1}{10}$ $(ii) P(A)=\text{P}(E_1)P(A\|E_1)+\text{P}(E_2)P(A\|E_2)$ $=\frac{40}{100}\times\frac{80}{100}+\frac{60}{100}\times\frac{10}{100}$ $=\frac{38}{100} \text{ or } \frac{19}{50}$ $(iii)(a) P(E_1 \mid A)=\frac{\text{P}(E_1)P(A\|E_1)}{P(A)}$ $=\frac{\frac{40}{100}\times\frac{80}{100}}{\frac{38}{100}}=\frac{16}{19}$ **OR** $(iii)(b) P(E_2 \mid A)=\frac{\text{P}(E_2)P(A\|E_2)}{P(A)}$ $=\frac{\frac{60}{100}\times\frac{10}{100}}{\frac{38}{100}}=\frac{3}{19}$	1 1 2 2

Q37. An engineer is designing a new metro rail network in a city.

Initially, two metro lines, Line A and Line B, each consisting of multiple stations are designed. The track for Line A is represented by $l_1 : \frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-3}{4}$, while the track for Line B is represented by $l_2 : \frac{x-1}{2} = \frac{y-3}{1} = \frac{z+2}{-3}$.

Based on the above information, answer the following questions :

(i) Find whether the two metro tracks are parallel. *1*

(ii) Solar panels are to be installed on the rooftop of the metro stations. Determine the equation of the line representing the placement of solar panels on the rooftop of Line A's stations, given that panels are to be positioned parallel to Line A's track (l_1) and pass through the point (1, – 2, – 3). *1*

(iii) (a) To connect the stations, a pedestrian pathway perpendicular to the two metro lines is to be constructed which passes through point (3, 2, 1). Determine the equation of the pedestrian walkway. *2*

OR

(iii) (b) Find the shortest distance between Line A and Line B. *2*

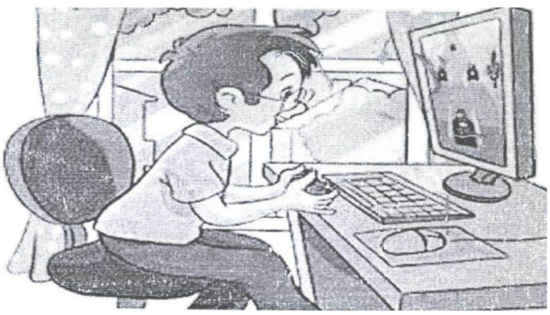
A37. $l_1 : \frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-3}{4}$; $l_2 : \frac{x-1}{2} = \frac{y-3}{1} = \frac{z+2}{-3}$

(i) Drs of l_1 are $<3,-2,4>$, drs of l_2 are $<2,1,-3>$

as drs are not proportional, hence l_1 is not parallel to l_2. 1

(ii) Equations of line parallel to l_1 and passing through $(1,-2,-3)$ is

$\frac{x-1}{3} = \frac{y+2}{-2} = \frac{z+3}{4}$ or $\vec{r} = \left(\hat{i} - 2\hat{j} - 3\hat{k}\right) + \lambda\left(3\hat{i} - 2\hat{j} + 4\hat{k}\right)$ 1

	$(iii)(a)$ The pathway is perpendicular to l_1 and l_2. $\therefore$ It is parallel to $\vec{b}_1 \times \vec{b}_2$ $\vec{b} = \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 4 \\ 2 & 1 & -3 \end{vmatrix} = 2\hat{i} + 17\hat{j} + 7\hat{k}$ $\therefore$ Equation of pathway is $\vec{r} = \left(3\hat{i} + 2\hat{j} + \hat{k}\right) + \lambda\left(2\hat{i} + 17\hat{j} + 7\hat{k}\right)$ **OR** $(iii)(b)$ $\vec{a}_1 = 2\hat{i} - \hat{j} + 3\hat{k}$, $\vec{a}_2 = \hat{i} + 3\hat{j} - 2\hat{k}$ $\vec{b}_1 = 3\hat{i} - 2\hat{j} + 4\hat{k}$, $\vec{b}_2 = 2\hat{i} + \hat{j} - 3\hat{k}$ $d = \left\| \frac{(\vec{a}_2 - \vec{a}_1).\left(\vec{b}_1 \times \vec{b}_2\right)}{\left\|\vec{b}_1 \times \vec{b}_2\right\|} \right\|$ $= \left\| \frac{\left(-\hat{i} + 4\hat{j} - 5\hat{k}\right).\left(2\hat{i} + 17\hat{j} + 7\hat{k}\right)}{\sqrt{4 + 289 + 49}} \right\|$ $= \frac{31}{\sqrt{342}}$	1 1 1 1
Q38.	During a heavy gaming session, the temperature of a student's laptop processor increases significantly. After the session, the processor begins to cool down, and the rate of cooling is proportional to the difference between the processor's temperature and the room temperature (25°C). Initially the processor's temperature is 85°C. The rate of cooling is defined by the equation $\frac{d}{dt}(T(t)) = -k(T(t) - 25)$, where T(t) represents the temperature of the processor at time t (in minutes) and k is a constant.  Based on the above information, answer the following questions : (i) Find the expression for temperature of processor, T(t) given that T(0) = 85°C. *2* (ii) How long will it take for the processor's temperature to reach 40°C ? Given that k = 0·03, $\log_e 4 = 1{\cdot}3863$. *2*	

A38.	$(i)\ \frac{dT}{dt}=-k(T-25)$ $\Rightarrow \frac{dT}{T-25}=-k\,dt$ $\Rightarrow \int \frac{dT}{T-25}=-k\int dt$ $\Rightarrow \log\lvert T-25\rvert=-kt+C \qquad ...(a)$ When $t=0, T=85$ $\Rightarrow \log 60=C$ Using in equation (a), $\log\lvert T-25\rvert=-kt+\log 60 \qquad ...(b)$ (ii) When $k=0.03, \log\lvert T-25\rvert=-0.03t+\log 60$ $\Rightarrow \log\left\lvert\frac{T-25}{60}\right\rvert=-0.03t$ $\Rightarrow T-25=60.e^{-0.03t}$ When $T=40, t=t_1$ $\Rightarrow \frac{15}{60}=e^{-0.03t_1}$ $\Rightarrow e^{-0.03t_1}=\frac{1}{4}\Rightarrow -0.03t_1=-\log 4$ $\Rightarrow t_1=\frac{\log 4}{0.03}=\frac{1.3863}{0.03}=46.21\text{m}$	 ½ 1 ½ 1 ½ ½